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David Sang, Graham Jones,
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Cambridge International AS and A Level

Physics

Coursebook

Second Edition

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org

First published 2010
Second edition 2014

Printed in the United Kingdom by Latimer Trend

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-69769-0 Paperback with CD-ROM for Windows® and MAC®

Example answers and all other end-of-chapter questions were written by the authors.

Contents

Introduction

This book covers the entire syllabus of Cambridge International Examinations AS and A Level Physics. It is designed to work with the syllabus that will be examined from 2016. It is in three parts:

- Chapters 1–16 and P1: the AS level content, covered in the first year of the course, including a chapter (P1) dedicated to the development of your practical skills
- Chapters 17–32 and P2: the remaining A level content, including a chapter (P2) dedicated to developing your ability to plan, analyse and evaluate practical investigations
- Appendices of useful formulae, a Glossary and an Index.

The main tasks of a textbook like this are to explain the various concepts of physics that you need to understand and to provide you with questions that will help you to test your understanding and prepare for your examinations. You will find a visual guide to the structure of each chapter and the features of this book on the next two pages.

When tackling questions, it is a good idea to make a first attempt without referring to the explanations in this Coursebook or to your notes. This will help to reveal any gaps in your understanding. By working out which concepts you find most challenging, and by spending more time to understand these concepts at an early stage, you will progress faster as the course continues.

The CD-ROM that accompanies this Coursebook includes answers with workings for all the questions in the book, as well as suggestions for revising and preparing for any examinations you take. There are also lists of recommended further reading, which in many cases will take you beyond the requirements of the syllabus, but which will help you deepen your knowledge and explain more of the background to the physics concepts covered in this Coursebook.

In your studies, you will find that certain key concepts come up again and again, and that these concepts form 'themes' that link the different areas of physics together. It will help you to progress and gain confidence in tackling problems if you take note of these themes. For this Coursebook, these key concepts include:

- Models of physical systems
- Testing predictions against evidence
- Mathematics as a language and problem-solving tool
- Matter, energy and waves
- Forces and fields

In this Coursebook, the mathematics has been kept to the minimum required by the Cambridge International Examinations AS and A Level Physics syllabus. If you are also studying mathematics, you may find that more advanced techniques such as calculus will help you with many aspects of physics.

Studying physics can be a stimulating and worthwhile experience. It is an international subject; no single country has a monopoly on the development of the ideas. It can be a rewarding exercise to discover how men and women from many countries have contributed to our knowledge and well-being, through their research into and application of the concepts of physics. We hope not only that this book will help you to succeed in your future studies and career, but also that it will stimulate your curiosity and fire your imagination. Today's students become the next generation of physicists and engineers, and we hope that you will learn from the past to take physics to ever-greater heights.

How to use this book

Each chapter begins with a short list of the facts and concepts that are explained in it.

Chapter 1: Kinematics – describing motion

Learning outcomes

You should be able to:

- define displacement, speed and velocity
- draw and interpret displacement–time graphs
- describe laboratory methods for determining speed
- use vector addition to add two or more vectors

There is a short context at the beginning of each chapter, containing an example of how the material covered in the chapter relates to the 'real world'.

Describing movement

Our eyes are good at detecting movement. We notice even quite small movements out of the corners of our eyes. It's important for us to be able to judge movement – think about crossing the road, cycling or driving, or catching a ball.

Figure 1.1 shows a way in which movement can be recorded on a photograph. This is a stroboscopic photograph of a boy juggling three balls. As he juggles, a bright lamp flashes several times a second so that the camera records the positions of the balls at equal intervals of time.

If we knew the time between flashes, we could measure the photograph and calculate the speed of a ball as it moves through the air.

Figure 1.1 This boy is juggling three balls. A stroboscopic lamp flashes at regular intervals; the camera is moved to one side at a steady rate to show separate images of the boy.

The text and illustrations describe and explain all of the facts and concepts that you need to know. The chapters, and often the content within them as well, are arranged in a similar sequence to your syllabus, but with AS and A Level content clearly separated into the two halves of the book.

Figure 13.3 or a similar graph of displacement against time illustrates the following important definitions about waves and wave motion:

- The distance of a point on the wave from its undisturbed position or equilibrium position is called the **displacement** x.
- The maximum displacement of any point on the wave from its undisturbed position is called the **amplitude** A. The amplitude of a wave on the sea is measured in units of distance, e.g. metres. The greater the amplitude of the wave, the louder the sound or the rougher the sea!
- The distance from any point on a wave to the next exactly similar point (e.g. crest to crest) is called the **wavelength** λ (the Greek letter lambda). The wavelength of a wave on the sea is measured in units of distance, e.g. metres.
- The time taken for one complete oscillation of a point in a wave is called the **period** T. It is the time taken for a point to move from one particular position and return to that same position, moving in the same direction. It is measured in units of time, e.g. seconds..
- The number of oscillations per unit time of a point in a wave is called its **frequency** f. For sound waves, the higher the frequency of a musical note, the higher is its pitch. Frequency is measured in hertz (Hz), where 1 Hz = one oscillation per second (1 kHz = 10^3 Hz and 1 MHz = 10^6 Hz). The frequency f of a wave is the reciprocal of the period T:

$$f = \frac{1}{T}$$

Waves are called **mechanical waves** if they need a substance (medium) through which to travel. Sound is one example of such a wave. Other cases are waves on strings, seismic waves and water waves (Figure 13.4).

Some properties of typical waves are given on page 183 in Table 13.1.

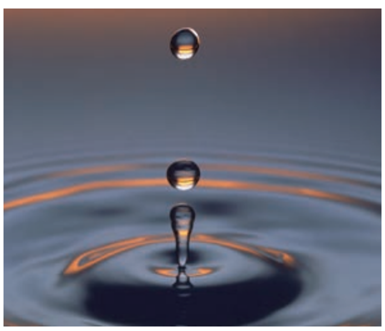


Figure 13.4 The impact of a droplet on the surface of a liquid creates a vibration, which in turn gives rise to waves on the surface.

QUESTION

1 Determine the wavelength and amplitude of each of the two waves shown in Figure 13.5.

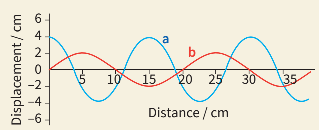


Figure 13.5 Two waves – for Question 1.

Questions throughout the text give you a chance to check that you have understood the topic you have just read about. You can find the answers to these questions on the CD-ROM.

BOX 13.1: Measuring frequency

You can measure the frequency of sound waves using a cathode-ray oscilloscope (c.r.o.). Figure 13.6 shows how.

A microphone is connected to the input of the c.r.o. Sound waves are captured by the microphone and converted into a varying voltage which has the same frequency as the sound waves. This voltage is displayed on the c.r.o. screen.

It is best to think of a c.r.o. as a voltmeter which is capable of displaying a rapidly varying voltage. To do this, its spot moves across the screen at a steady speed, set by the time-base control. At the same time, the spot moves up and down according to the voltage of the input.

Hence the display on the screen is a graph of the varying voltage, with time on the (horizontal) x-axis. If we know the horizontal scale, we can determine the period and hence the frequency of the sound wave. Worked example **1** shows how to do this. (In Chapter 15 we will look at one method of measuring the wavelength of sound waves.)

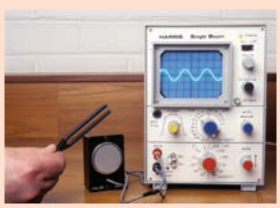


Figure 13.6 Measuring the frequency of sound waves from a tuning fork.

This book does not contain detailed instructions for doing particular experiments, but you will find background information about the practical work you need to do in these Boxes. There are also two chapters, P1 and P2, which provide detailed information about the practical skills you need to develop during your course.

Important equations and other facts are shown in highlight boxes.

For an object of mass m travelling at a speed v, we have:

kinetic energy = $\frac{1}{2}$ × mass × speed^2

$$E_k = \frac{1}{2}mv^2$$

Wherever you need to know how to use a formula to carry out a calculation, there are worked example boxes to show you how to do this.

WORKED EXAMPLE

1 In Figure 6.5, trolley A of mass 0.80 kg travelling at a velocity of 3.0 m s^{-1} collides head-on with a stationary trolley B. Trolley B has twice the mass of trolley A. The trolleys stick together and have a common velocity of 1.0 m s^{-1} after the collision. Show that momentum is conserved in this collision.

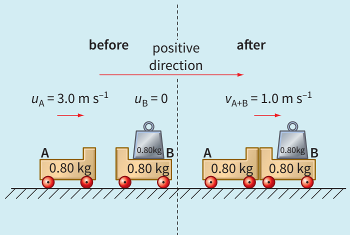


Figure 6.5 The state of trolleys A and B, before and after the collision.

Step 1 Make a sketch using the information given in the question. Notice that we need two diagrams to show the situations, one before and one after the collision. Similarly, we need two calculations – one for the momentum of the trolleys before the collision and one for their momentum after the collision.

Step 2 Calculate the momentum before the collision:

momentum of trolleys before collision

$= m_A \times u_A + m_B \times u_B$

$= (0.80 \times 3.0) + 0$

$= 2.4\,\text{kg m s}^{-1}$

Trolley B has no momentum before the collision, because it is not moving.

Step 3 Calculate the momentum after the collision:

momentum of trolleys after collision

$= (m_A + m_B) \times v_{A+B}$

$= (0.80 + 1.60) \times 1.0$

$= 2.4\,\text{kg m s}^{-1}$

So, both before and after the collision, the trolleys have a combined momentum of 2.4 kg m s^{-1}. Momentum has been conserved.

Key words are highlighted in the text when they are first introduced.

The metre, kilogram and second are three of the seven SI base units. These are defined with great precision so that every standards laboratory can reproduce them correctly.

You will also find definitions of these words in the Glossary.

base units Defined units of the SI system from which all other units are derived.

There is a summary of key points at the end of each chapter. You might find this helpful when you are revising.

Summary

- Forces are vector quantities that can be added by means of a vector triangle. Their resultant can be determined using trigonometry or by scale drawing.
- Vectors such as forces can be resolved into components. Components at right angles to one another can be treated independently of one another. For a force F at an angle θ to the x-direction, the components are:

  x-direction: $F\cos\theta$

  y-direction: $F\sin\theta$
- The moment of a force = force × perpendicular distance of the pivot from the line of action of the force.
- The principle of moments states that, for any object that is in equilibrium, the sum of the clockwise moments about any point provided by the forces acting on the object equals the sum of the anticlockwise moments about that same point.
- A couple is a pair of equal, parallel but opposite forces whose effect is to produce a turning effect on a body without giving it linear acceleration.

  torque of a couple = one of the forces × perpendicular distance between the forces
- For an object to be in equilibrium, the resultant force acting on the object must be zero and the resultant moment must be zero.

Questions at the end of each chapter begin with shorter answer questions, then move on to more demanding exam-style questions, some of which may require use of knowledge from previous chapters. Answers to these questions can be found on the CD–ROM.

1 Figure 15.19 shows a stationary wave on a string.

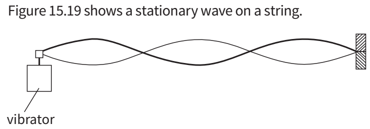


Figure 15.19 For End-of-chapter Question 1.

a On a copy of Figure 15.19, label one **node** (N) and one **antinode** (A). [1]

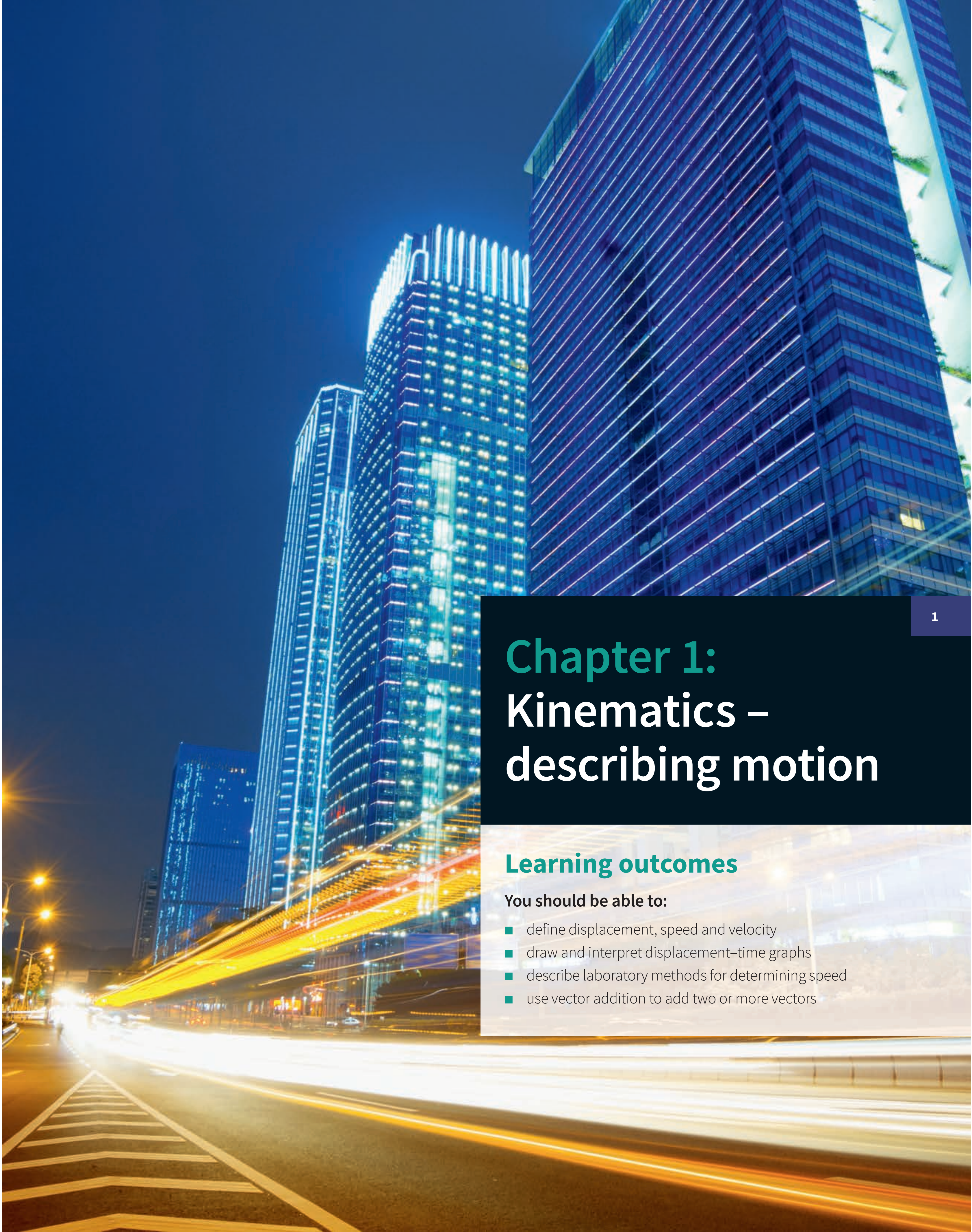
b Mark on your diagram the wavelength of the standing wave and label it λ. [1]

c The frequency of the vibrator is doubled. Describe the changes in the standing wave pattern. [1]

2 A tuning fork which produces a note of 256 Hz is placed above a tube which is nearly filled with water. The water level is lowered until resonance is first heard.

a Explain what is meant by the term **resonance**. [1]

b The length of the column of air above the water when resonance is first heard is 31.2 cm. Calculate the speed of the sound wave. [2]



Chapter 1: Kinematics – describing motion

Learning outcomes

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- define displacement, speed and velocity
- draw and interpret displacement–time graphs
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Describing movement

Our eyes are good at detecting movement. We notice even quite small movements out of the corners of our eyes. It's important for us to be able to judge movement – think about crossing the road, cycling or driving, or catching a ball.

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If we knew the time between flashes, we could measure the photograph and calculate the speed of a ball as it moves through the air.

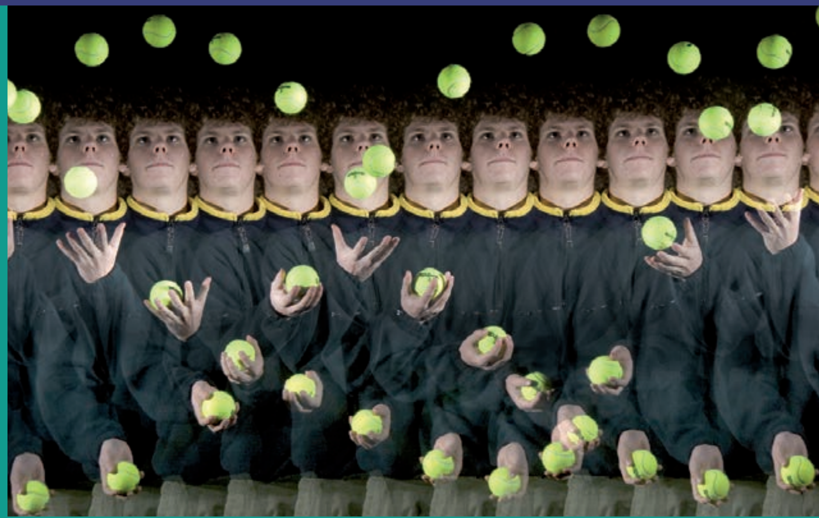


Figure 1.1 This boy is juggling three balls. A stroboscopic lamp flashes at regular intervals; the camera is moved to one side at a steady rate to show separate images of the boy.

Speed

We can calculate the average speed of something moving if we know the distance it moves and the time it takes:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

In symbols, this is written as:

$$v = \frac{d}{t}$$

where v is the average speed and d is the distance travelled in time t. The photograph (Figure 1.2) shows Ethiopia's Kenenisa Bekele posing next to the scoreboard after breaking the world record in a men's 10 000 metres race. The time on the clock in the photograph enables us to work out his average speed.

If the object is moving at a constant speed, this equation will give us its speed during the time taken. If its speed is changing, then the equation gives us its **average speed**. Average speed is calculated over a period of time.

Figure 1.2 Ethiopia's Kenenisa Bekele set a new world record for the 10 000 metres race in 2005.

If you look at the speedometer in a car, it doesn't tell you the car's average speed; rather, it tells you its speed at the instant when you look at it. This is the car's **instantaneous speed**.

QUESTION

1 Look at Figure 1.2. The runner ran 10 000 m, and the clock shows the total time taken. Calculate his average speed during the race.

Units

In the Système Internationale d'Unités (the SI system), distance is measured in metres (m) and time in seconds (s). Therefore, speed is in metres per second. This is written as $m\,s^{-1}$ (or as m/s). Here, s^{-1} is the same as 1/s, or 'per second'.

There are many other units used for speed. The choice of unit depends on the situation. You would probably give the speed of a snail in different units from the speed of a racing car. Table 1.1 includes some alternative units of speed.

Note that in many calculations it is necessary to work in SI units ($m\,s^{-1}$).

$m\,s^{-1}$	metres per second
$cm\,s^{-1}$	centimetres per second
$km\,s^{-1}$	kilometres per second
$km\,h^{-1}$ or km/h	kilometres per hour
mph	miles per hour

Table 1.1 Units of speed.

QUESTIONS

2 Here are some units of speed:

$m\,s^{-1}$ $mm\,s^{-1}$ $km\,s^{-1}$ $km\,h^{-1}$

Which of these units would be appropriate when stating the speed of each of the following?

a a tortoise

b a car on a long journey

c light

d a sprinter.

3 A snail crawls 12 cm in one minute. What is its average speed in $mm\,s^{-1}$?

Determining speed

You can find the speed of something moving by measuring the time it takes to travel between two fixed points. For example, some motorways have emergency telephones every 2000 m. Using a stopwatch you can time a car over this distance. Note that this can only tell you the car's average speed between the two points. You cannot tell whether it was increasing its speed, slowing down, or moving at a constant speed.

BOX 1.1: Laboratory measurements of speed

Here we describe four different ways to measure the speed of a trolley in the laboratory as it travels along a straight line. Each can be adapted to measure the speed of other moving objects, such as a glider on an air track, or a falling mass.

Measuring speed using two light gates

The leading edge of the card in Figure 1.3 breaks the light beam as it passes the first light gate. This starts the timer. The timer stops when the front of the card breaks the second beam. The trolley's speed is calculated from the time interval and the distance between the light gates.

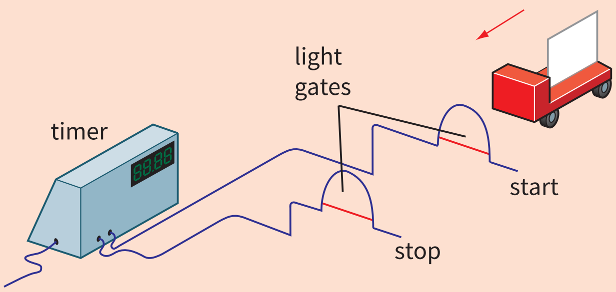


Figure 1.3 Using two light gates to find the average speed of a trolley.

Measuring speed using one light gate

The timer in Figure 1.4 starts when the leading edge of the card breaks the light beam. It stops when the trailing edge passes through. In this case, the time shown is the time taken for the trolley to travel a distance equal to the length of the card. The computer software can calculate the speed directly by dividing the distance by the time taken.

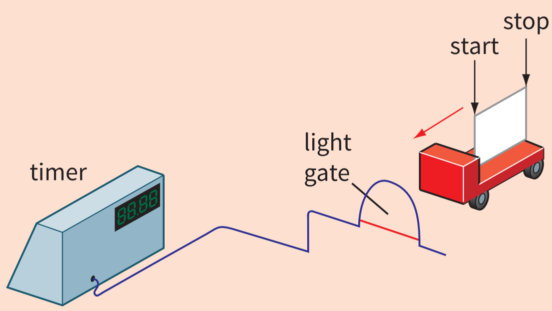


Figure 1.4 Using a single light gate to find the average speed of a trolley.

Measuring speed using a ticker-timer

The ticker-timer (Figure 1.5) marks dots on the tape at regular intervals, usually s (i.e. 0.02 s). (This is because it works with alternating current, and in most countries the frequency of the alternating mains is 50 Hz.) The pattern of dots acts as a record of the trolley's movement.

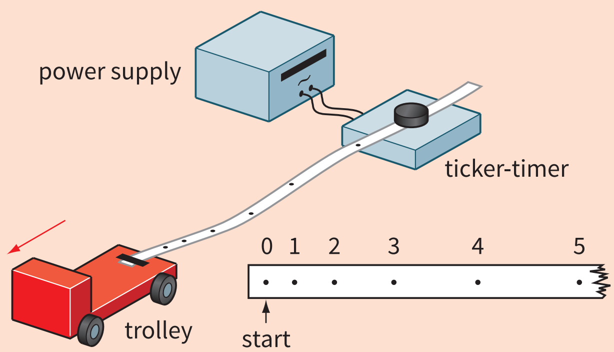


Figure 1.5 Using a ticker-timer to investigate the motion of a trolley.

BOX 1.1: Laboratory measurements of speed (continued)

Start by inspecting the tape. This will give you a description of the trolley's movement. Identify the start of the tape. Then look at the spacing of the dots:

- even spacing – constant speed
- increasing spacing – increasing speed.

Now you can make some measurements. Measure the distance of every fifth dot from the start of the tape. This will give you the trolley's distance at intervals of 0.1 s. Put the measurements in a table and draw a distance–time graph.

Measuring speed using a motion sensor

The motion sensor (Figure 1.6) transmits regular pulses of ultrasound at the trolley. It detects the reflected waves and determines the time they took for the trip to the trolley and back. From this, the computer can deduce the distance to the trolley from the motion sensor. It can generate a distance–time graph. You can determine the speed of the trolley from this graph.

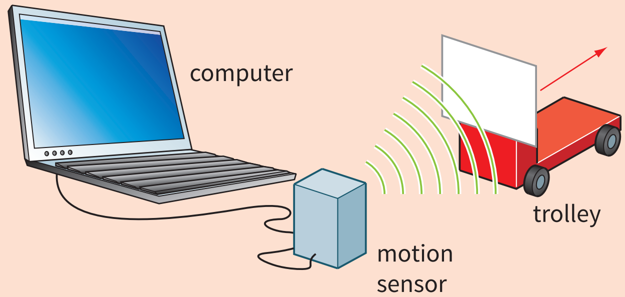


Figure 1.6 Using a motion sensor to investigate the motion of a trolley.

Choosing the best method

Each of these methods for finding the speed of a trolley has its merits. In choosing a method, you might think about the following points:

- Does the method give an average value of speed or can it be used to give the speed of the trolley at different points along its journey?
- How precisely does the method measure time – to the nearest millisecond?
- How simple and convenient is the method to set up in the laboratory?

QUESTIONS

4 A trolley with a 5.0 cm long card passed through a single light gate. The time recorded by a digital timer was 0.40 s. What was the average speed of the trolley in $m\,s^{-1}$?

5 Figure 1.7 shows two ticker-tapes. Describe the motion of the trolleys which produced them.

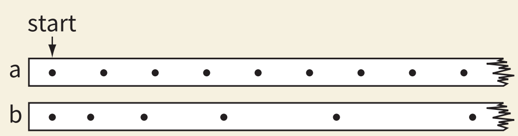


Figure 1.7 Two ticker-tapes; for Question 5.

6 Four methods for determining the speed of a moving trolley have been described. Each could be adapted to investigate the motion of a falling mass. Choose two methods which you think would be suitable, and write a paragraph for each to say how you would adapt it for this purpose.

Distance and displacement, scalar and vector

In physics, we are often concerned with the distance moved by an object in a particular direction. This is called its **displacement**. Figure 1.8 illustrates the difference between distance and displacement. It shows the route followed by walkers as they went from town A to town C. Their winding route took them through town B, so that they covered a total distance of 15 km. However, their displacement was much less than this. Their finishing position was just 10 km from where they started. To give a complete statement of their displacement, we need to give both distance and direction:

displacement = 10 km 30° E of N

Displacement is an example of a **vector quantity**. A vector quantity has both magnitude (size) and direction. Distance, on the other hand, is a **scalar quantity**. Scalar quantities have magnitude only.

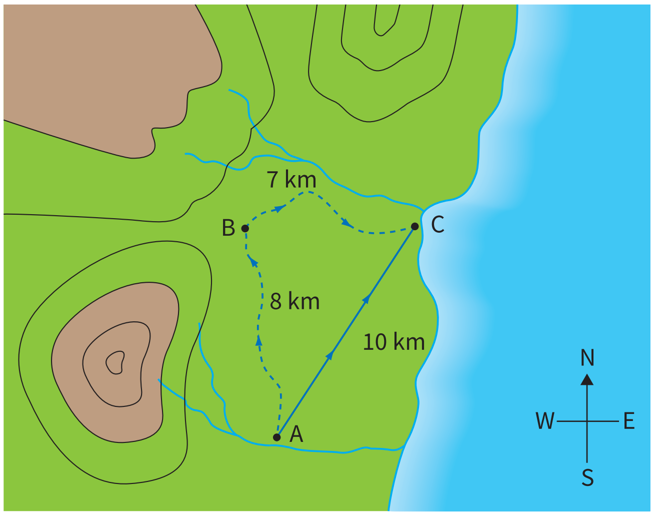


Figure 1.8 If you go on a long walk, the distance you travel will be greater than your displacement. In this example, the walkers travel a distance of 15 km, but their displacement is only 10 km, because this is the distance from the start to the finish of their walk.

Speed and velocity

It is often important to know both the speed of an object and the direction in which it is moving. Speed and direction are combined in another quantity, called **velocity**. The velocity of an object can be thought of as its speed in a particular direction. So, like displacement, velocity is a **vector** quantity. Speed is the corresponding scalar quantity, because it does not have a direction. So, to give the velocity of something, we have to state the direction in which it is moving. For example, an aircraft flies with a velocity of $300\,\mathrm{m\,s^{-1}}$ due north. Since velocity is a vector quantity, it is defined in terms of displacement:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

Alternatively, we can say that velocity is the rate of change of an object's displacement. From now on, you need to be clear about the distinction between velocity and speed, and between displacement and distance. Table 1.2 shows the standard symbols and units for these quantities.

Quantity	Symbol for quantity	Symbol for unit
distance	d	m
displacement	s, x	m
time	t	s
speed, velocity	v	$\mathrm{m\,s^{-1}}$

Table 1.2 Standard symbols and units. (Take care not to confuse italic s for displacement with s for seconds. Notice also that v is used for both speed and velocity.)

QUESTION

7 Which of these gives speed, velocity, distance or displacement? (Look back at the definitions of these quantities.)

a The ship sailed south-west for 200 miles.

b I averaged 7 mph during the marathon.

c The snail crawled at $2\,\mathrm{mm\,s^{-1}}$ along the straight edge of a bench.

d The sales representative's round trip was 420 km.

Speed and velocity calculations

We can write the equation for velocity in symbols:

$$v = \frac{s}{t}$$

$$v = \frac{\Delta s}{\Delta t}$$

The word equation for velocity is:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

Note that we are using Δs to mean 'change in displacement s'. The symbol Δ, Greek letter delta, means 'change in'. It does not represent a quantity (in the way that s does); it is simply a convenient way of representing a change in a quantity. Another way to write Δs would be $s_2 - s_1$, but this is more time-consuming and less clear.

The equation for velocity, $v = \frac{\Delta s}{\Delta t}$, can be rearranged as follows, depending on which quantity we want to determine:

$$\text{change in displacement } \Delta s = v \times \Delta t$$

$$\text{change in time } \Delta t = \frac{\Delta s}{v}$$

Note that each of these equations is balanced in terms of units. For example, consider the equation for displacement. The units on the right-hand side are $\mathrm{m\,s^{-1}} \times \mathrm{s}$, which simplifies to m, the correct unit for displacement.

Note also that we can, of course, use the same equations to find speed and distance, that is:

$$v = \frac{d}{t}$$

$$\text{distance } d = v \times t$$

$$\text{time } t = \frac{d}{v}$$

WORKED EXAMPLES

1 A car is travelling at 15 m s^{-1}. How far will it travel in 1 hour?

Step 1 It is helpful to start by writing down what you know and what you want to know:

$v = 15\,\text{m s}^{-1}$

$t = 1\,\text{h} = 3600\,\text{s}$

$d = ?$

Step 2 Choose the appropriate version of the equation and substitute in the values. Remember to include the units:

$d = v \times t$

$= 15 \times 3600$

$= 5.4 \times 10^4\,\text{m}$

$= 54\,\text{km}$

The car will travel 54 km in 1 hour.

2 The Earth orbits the Sun at a distance of 150 000 000 km. How long does it take light from the Sun to reach the Earth?
(Speed of light in space = $3.0 \times 10^8\,\text{m s}^{-1}$.)

Step 1 Start by writing what you know. Take care with units; it is best to work in m and s. You need to be able to express numbers in scientific notation (using powers of 10) and to work with these on your calculator.

$v = 3.0 \times 10^8\,\text{m s}^{-1}$

$d = 150\,000\,000\,\text{km}$

$= 150\,000\,000\,000\,\text{m}$

$= 1.5 \times 10^{11}\,\text{m}$

Step 2 Substitute the values in the equation for time:

$$t = \frac{d}{v} = \frac{1.5 \times 10^{11}}{3.0 \times 10^8} = 500\,\text{s}$$

Light takes 500 s (about 8.3 minutes) to travel from the Sun to the Earth.

Hint: *When using a calculator, to calculate the time t, you press the buttons in the following sequence:*

[1.5] [EXP] [11] [÷] [3] [EXP] [8]

or

[1.5] [×10^n] [11] [÷] [3] [×10^n] [8]

Making the most of units

In Worked example 1 and Worked example 2, units have been omitted in intermediate steps in the calculations. However, at times it can be helpful to include units as this can be a way of checking that you have used the correct equation; for example, that you have not divided one quantity by another when you should have multiplied them. The units of an equation must be balanced, just as the numerical values on each side of the equation must be equal.

If you take care with units, you should be able to carry out calculations in non-SI units, such as kilometres per hour, without having to convert to metres and seconds.

For example, how far does a spacecraft travelling at 40 000 km h^{-1} travel in one day? Since there are 24 hours in one day, we have:

$$\text{distance travelled} = 40\,000\,\text{km h}^{-1} \times 24\,\text{h} = 960\,000\,\text{km}$$

QUESTIONS

8 A submarine uses sonar to measure the depth of water below it. Reflected sound waves are detected 0.40 s after they are transmitted. How deep is the water? (Speed of sound in water = 1500 m s^{-1}.)

9 The Earth takes one year to orbit the Sun at a distance of 1.5×10^{11} m. Calculate its speed. Explain why this is its average speed and not its velocity.

Displacement–time graphs

We can represent the changing position of a moving object by drawing a displacement–time graph. The gradient (slope) of the graph is equal to its velocity (Figure 1.9). The steeper the slope, the greater the velocity. A graph like this can also tell us if an object is moving forwards or backwards. If the gradient is negative, the object's velocity is negative – it is moving backwards.

Deducing velocity from a displacement–time graph

A toy car moves along a straight track. Its displacement at different times is shown in Table 1.3. This data can be used to draw a displacement–time graph from which we can deduce the car's velocity.

Displacement / m	1.0	3.0	5.0	7.0	7.0	7.0
Time / s	0.0	1.0	2.0	3.0	4.0	5.0

Table 1.3 Displacement (s) and time (t) data for a toy car.

The straight line shows that the object's velocity is constant.

The slope shows which object is moving faster. The steeper the slope, the greater the velocity.

The slope of this graph is 0. The displacement s is not changing. Hence the velocity $v = 0$. The object is stationary.

The slope of this graph suddenly becomes negative. The object is moving back the way it came. Its velocity v is negative after time T.

This displacement–time graph is curved. The slope is changing. This means that the object's velocity is changing – this is considered in Chapter 2.

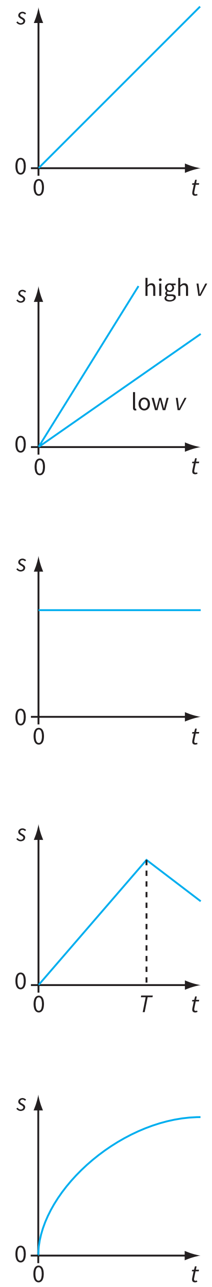


Figure 1.9 The slope of a displacement–time (s–t) graph tells us how fast an object is moving.

It is useful to look at the data first, to see the pattern of the car's movement. In this case, the displacement increases steadily at first, but after 3.0 s it becomes constant. In other words, initially the car is moving at a steady velocity, but then it stops.

Now we can plot the displacement–time graph (Figure 1.11).

We want to work out the velocity of the car over the first 3.0 seconds. We can do this by working out the gradient of the graph, because:

velocity = gradient of displacement–time graph

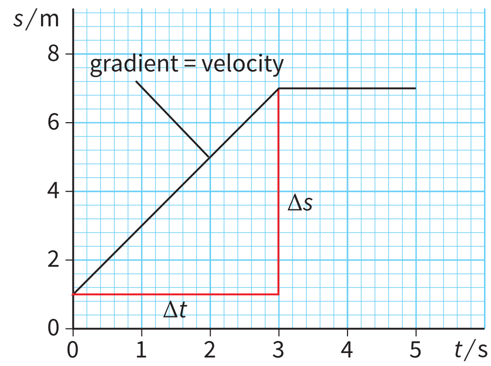


Figure 1.11 Displacement–time graph for a toy car; data as shown in Table 1.3.

We draw a right-angled triangle as shown. To find the car's velocity, we divide the change in displacement by the change in time. These are given by the two sides of the triangle labelled Δs and Δt.

$$\text{velocity } v = \frac{\text{change in displacement}}{\text{time taken}}$$

$$v = \frac{\Delta s}{\Delta t}$$

$$v = \frac{(7.0 - 1.0)}{(3.0 - 0)} = \frac{6.0}{3.0} = 2.0\,\text{m s}^{-1}$$

If you are used to finding the gradient of a graph, you may be able to reduce the number of steps in this calculation.

QUESTIONS

10 The displacement–time sketch graph in Figure 1.10 represents the journey of a bus. What does the graph tell you about the journey?

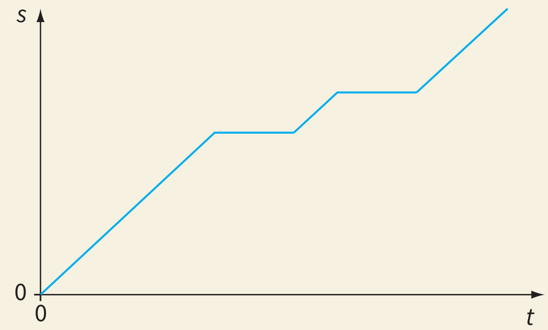


Figure 1.10 For Question 10.

11 Sketch a displacement–time graph to show your motion for the following event. You are walking at a constant speed across a field after jumping off a gate. Suddenly you see a bull and stop. Your friend says there's no danger, so you walk on at a reduced constant speed. The bull bellows, and you run back to the gate. Explain how each section of the walk relates to a section of your graph.

12 Table 1.4 shows the displacement of a racing car at different times as it travels along a straight track during a speed trial.

a Determine the car's velocity.

b Draw a displacement–time graph and use it to find the car's velocity.

Displacement / m	0	85	170	255	340
Time / s	0	1.0	2.0	3.0	4.0

Table 1.4 Displacement (s) and time (t) data for Question 12.

13 An old car travels due south. The distance it travels at hourly intervals is shown in Table 1.5.

a Draw a distance–time graph to represent the car's journey.

b From the graph, deduce the car's speed in km h^{-1} during the first three hours of the journey.

c What is the car's average speed in km h^{-1} during the whole journey?

Time / h	**Distance / km**
0	0
1	23
2	46
3	69
4	84

Table 1.5 Data for Question 13.

Combining displacements

The walkers shown in Figure 1.12 are crossing difficult ground. They navigate from one prominent point to the next, travelling in a series of straight lines. From the map, they can work out the distance that they travel and their displacement from their starting point:

distance travelled = 25 km

(Lay thread along route on map; measure thread against map scale.)

displacement = 15 km north-east

(Join starting and finishing points with straight line; measure line against scale.)

A map is a scale drawing. You can find your displacement by measuring the map. But how can you **calculate** your displacement? You need to use ideas from geometry and trigonometry. Worked examples 3 and 4 show how.

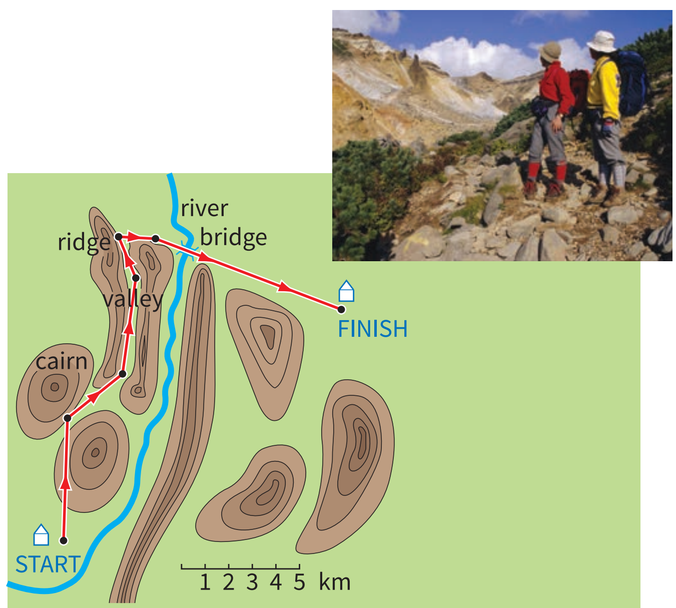


Figure 1.12 In rough terrain, walkers head straight for a prominent landmark.

WORKED EXAMPLES

3 A spider runs along two sides of a table (Figure 1.13). Calculate its final displacement.

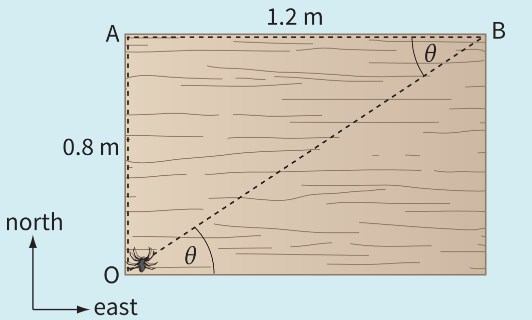


Figure 1.13 The spider runs a distance of 2.0 m, but what is its displacement?

Step 1 Because the two sections of the spider's run (OA and AB) are at right angles, we can **add** the two displacements using Pythagoras's theorem:

$OB^2 = OA^2 + AB^2$

$= 0.8^2 + 1.2^2 = 2.08$

$OB = \sqrt{2.08} = 1.44\,\text{m} \approx 1.4\,\text{m}$

Step 2 Displacement is a vector. We have found the **magnitude** of this vector, but now we have to find its direction. The angle θ is given by:

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{0.8}{1.2}$

$= 0.667$

$\theta = \tan^{-1}(0.667)$

$= 33.7° \approx 34°$

So the spider's displacement is 1.4 m at an angle of 34° north of east.

4 An aircraft flies 30 km due east and then 50 km north-east (Figure 1.14). Calculate the final displacement of the aircraft.

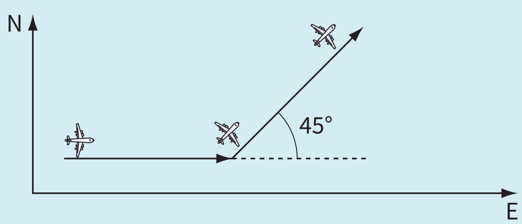


Figure 1.14 What is the aircraft's final displacement?

Here, the two displacements are not at 90° to one another, so we can't use Pythagoras's theorem. We can solve this problem by making a scale drawing, and measuring the final displacement. (However, you could solve the same problem using trigonometry.)

Step 1 Choose a suitable scale. Your diagram should be reasonably large; in this case, a scale of 1 cm to represent 5 km is reasonable.

Step 2 Draw a line to represent the first vector. North is at the top of the page. The line is 6 cm long, towards the east (right).

Step 3 Draw a line to represent the second vector, starting at the end of the first vector. The line is 10 cm long, and at an angle of 45° (Figure 1.15).

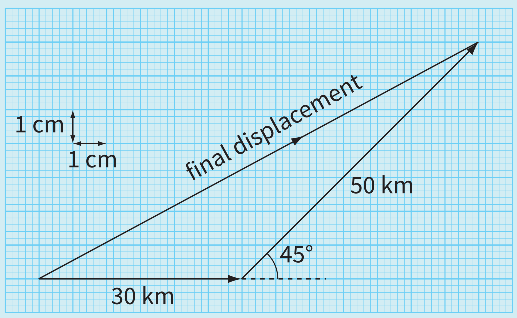


Figure 1.15 Scale drawing for Worked example 4. Using graph paper can help you to show the vectors in the correct directions.

Step 4 To find the final displacement, join the start to the finish. You have created a **vector triangle**. Measure this displacement vector, and use the scale to convert back to kilometres:

length of vector = 14.8 cm

final displacement = 14.8 × 5 = 74 km

Step 5 Measure the angle of the final displacement vector:

angle = 28° N of E

Therefore the aircraft's final displacement is 74 km at 28° north of east.

QUESTIONS

14 You walk 3.0 km due north, and then 4.0 km due east.

a Calculate the total distance in km you have travelled.

b Make a scale drawing of your walk, and use it to find your final displacement. Remember to give both the magnitude and the direction.

c Check your answer to part **b** by calculating your displacement.

15 A student walks 8.0 km south-east and then 12 km due west.

a Draw a vector diagram showing the route. Use your diagram to find the total displacement. Remember to give the scale on your diagram and to give the direction as well as the magnitude of your answer.

b Calculate the resultant displacement. Show your working clearly.

This process of adding two displacements together (or two or more of any type of vector) is known as **vector addition**. When two or more vectors are added together, their combined effect is known as the **resultant** of the vectors.

Combining velocities

Velocity is a vector quantity and so two velocities can be combined by vector addition in the same way that we have seen for two or more displacements.

Imagine that you are attempting to swim across a river. You want to swim directly across to the opposite bank, but the current moves you sideways at the same time as you are swimming forwards. The outcome is that you will end up on the opposite bank, but downstream of your intended landing point. In effect, you have two velocities:

- the velocity due to your swimming, which is directed straight across the river
- the velocity due to the current, which is directed downstream, at right angles to your swimming velocity.

These combine to give a **resultant** (or net) velocity, which will be diagonally downstream. In order to swim directly across the river, you would have to aim upstream. Then your resultant velocity could be directly across the river.

WORKED EXAMPLE

5 An aircraft is flying due north with a velocity of 200 m s^{-1}. A side wind of velocity 50 m s^{-1} is blowing due east. What is the aircraft's resultant velocity (give the magnitude and direction)?

Here, the two velocities are at 90°. A sketch diagram and Pythagoras's theorem are enough to solve the problem.

Step 1 Draw a sketch of the situation – this is shown in Figure 1.16a.

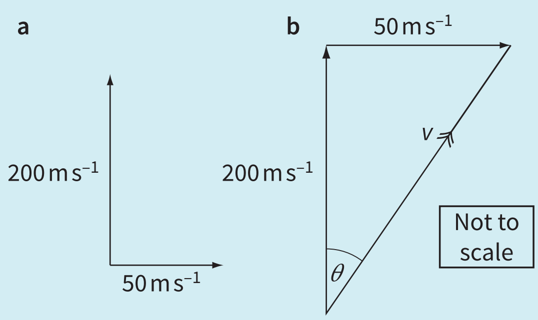


Figure 1.16 Finding the resultant of two velocities – for Worked example 5.

Step 2 Now sketch a vector triangle. Remember that the second vector starts where the first one ends. This is shown in Figure 1.16b.

Step 3 Join the start and end points to complete the triangle.

Step 4 Calculate the magnitude of the resultant vector v (the hypotenuse of the right-angled triangle).

$$v^2 = 200^2 + 50^2 = 40\,000 + 2500 = 42\,500$$

$$v = \sqrt{42\,500} \approx 206 \text{ m s}^{-1}$$

Step 5 Calculate the angle θ:

$$\tan\theta = \frac{50}{200}$$

$$= 0.25$$

$$\theta = \tan^{-1}(0.25) \approx 14°$$

So the aircraft's resultant velocity is 206 m s^{-1} at 14° east of north.

QUESTIONS

16 A swimmer can swim at 2.0 m s^{-1} in still water. She aims to swim directly across a river which is flowing at 0.80 m s^{-1}. Calculate her resultant velocity. (You must give both the magnitude and the direction.)

17 A stone is thrown from a cliff and strikes the surface of the sea with a vertical velocity of 18 m s^{-1} and a horizontal velocity v. The resultant of these two velocities is 25 m s^{-1}.

a Draw a vector diagram showing the two velocities and the resultant.

b Use your diagram to find the value of v.

c Use your diagram to find the angle between the stone and the vertical as it strikes the water.

Summary

- Displacement is the distance travelled in a particular direction.
- Velocity is defined by the word equation

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

The gradient of a displacement–time graph is equal to velocity:

$$\text{velocity} = \frac{\Delta s}{\Delta t}$$

- Distance and speed are scalar quantities. A scalar quantity has only magnitude.
- Displacement and velocity are vector quantities. A vector quantity has both magnitude and direction.
- Vector quantities may be combined by vector addition to find their resultant.

End-of-chapter questions

1 A car travels one complete lap around a circular track at a constant speed of $120\,km\,h^{-1}$.

a If one lap takes 2.0 minutes, show that the length of the track is 4.0 km. [2]

b Explain why values for the **average speed** and **average velocity** are different. [1]

c Determine the magnitude of the displacement of the car in a time of 1.0 minute. [2]
(The circumference of a circle = $2\pi R$, where R is the radius of the circle.)

2 A boat leaves point A and travels in a straight line to point B (Figure 1.17). The journey takes 60 s.

Calculate:

a the distance travelled by the boat [2]

b the total displacement of the boat [2]

c the average velocity of the boat. [2]

Remember that each vector quantity must be given a direction as well as a magnitude.

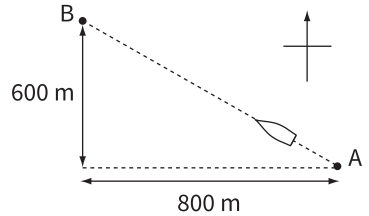


Figure 1.17 For End-of-chapter Question 2.

3 A boat travels at $2.0\,m\,s^{-1}$ east towards a port, 2.2 km away. When the boat reaches the port, the passengers travel in a car due north for 15 minutes at $60\,km\,h^{-1}$.

Calculate:

a the total distance travelled [2]

b the total displacement [2]

c the total time taken [2]

d the average speed in $m\,s^{-1}$ [2]

e the magnitude of the average velocity. [2]

4 A river flows from west to east with a constant velocity of $1.0\,m\,s^{-1}$. A boat leaves the south bank heading due north at $2.40\,m\,s^{-1}$. Find the resultant velocity of the boat. [2]

5 a Define **displacement**. [1]

b Use the definition of displacement to explain how it is possible for an athlete to run round a track yet have no displacement. [2]

6 A girl is riding a bicycle at a constant velocity of $3.0\,m\,s^{-1}$ along a straight road. At time $t = 0$, she passes a boy sitting on a stationary bicycle. At time $t = 0$, the boy sets off to catch up with the girl. His velocity increases from time $t = 0$ until $t = 5.0\,s$, when he has covered a distance of 10 m. He then continues at a constant velocity of $4.0\,m\,s^{-1}$.

a Draw the displacement–time graph for the girl from $t = 0$ to 12 s. [1]

b On the same graph axes, draw the displacement–time graph for the boy. [2]

c Using your graph, determine the value of t when the boy catches up with the girl. [1]

7 A student drops a small black sphere alongside a vertical scale marked in centimetres. A number of flash photographs of the sphere are taken at 0.1 s intervals, as shown in Figure 1.18. The first photograph is taken with the sphere at the top at time t = 0 s.

a Explain how Figure 1.18 shows that the sphere reaches a constant speed. [2]

b Determine the constant speed reached by the sphere. [2]

c Determine the distance that the sphere has fallen when t = 0.8 s. [2]

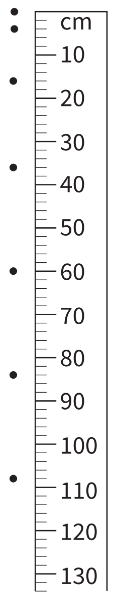


Figure 1.18 For End-of-chapter Question 7.

8 **a** State **one** difference between a scalar quantity and a vector quantity and give an example of each. [3]

b A plane has an air speed of 500 km h^{-1} due north. A wind blows at 100 km h^{-1} from east to west. Draw a vector diagram to calculate the resultant velocity of the plane. Give the direction of travel of the plane with respect to north. [4]

c The plane flies for 15 minutes. Calculate the displacement of the plane in this time. [1]

9 A small aircraft for one person is used on a short horizontal flight. On its journey from A to B, the resultant velocity of the aircraft is 15 m s^{-1} in a direction 60° east of north and the wind velocity is 7.5 m s^{-1} due north (Figure 1.19).

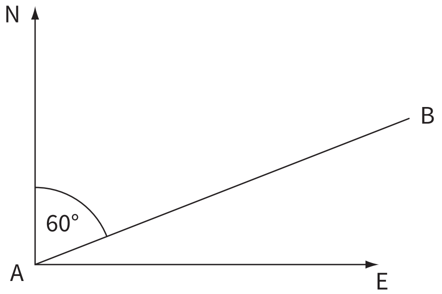


Figure 1.19 For End-of-chapter Question 9.

a Show that for the aircraft to travel from A to B it should be pointed due east. [2]

b After flying 5 km from A to B, the aircraft returns along the same path from B to A with a resultant velocity of 13.5 m s^{-1}. Assuming that the time spent at B is negligible, calculate the average speed for the complete journey from A to B and back to A. [3]

Chapter 2: Accelerated motion

Learning outcomes

You should be able to:

- define acceleration
- draw and interpret velocity–time graphs
- derive and use the equations of uniformly accelerated motion
- describe a method for determining the acceleration due to gravity, g
- explain projectile motion in terms of horizontal and vertical components of motion

Quick off the mark

The cheetah (Figure 2.1) has a maximum speed of over 30 m s^{-1} (108 km/h). From a standing start a cheetah can reach 20 m s^{-1} in just three or four strides, taking only two seconds.

A car cannot increase its speed as rapidly but on a long straight road it can easily travel faster than a cheetah.

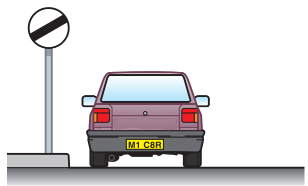
Figure 2.1 The cheetah is the world's fastest land animal. Its acceleration is impressive, too.

The meaning of acceleration

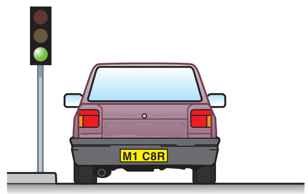
In everyday language, the term **accelerating** means 'speeding up'. Anything whose speed is increasing is accelerating. Anything whose speed is decreasing is decelerating.

To be more precise in our definition of acceleration, we should think of it as **changing velocity**. Any object whose speed is changing or which is changing its **direction** has acceleration. Because acceleration is linked to velocity in this way, it follows that it is a **vector** quantity.

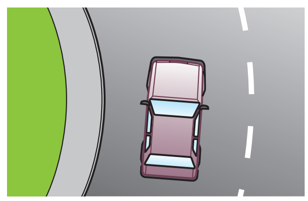
Some examples of objects accelerating are shown in Figure 2.2.



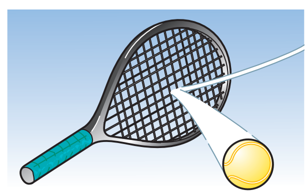
A car speeding up as it leaves the town. The driver presses on the accelerator pedal to increase the car's velocity.



A car setting off from the traffic lights. There is an instant when the car is both stationary **and** accelerating. Otherwise it would not start moving.



A car travelling round a bend at a steady speed. The car's speed is constant, but its velocity is changing as it changes direction.



A ball being hit by a tennis racket. Both the ball's speed and direction are changing. The ball's velocity changes.

A stone dropped over a cliff. Gravity makes the stone go faster and faster. The stone accelerates as it falls.

Figure 2.2 Examples of objects accelerating.

Calculating acceleration

The acceleration of something indicates the rate at which its velocity is changing. Language can get awkward here. Looking at the sprinter in Figure 2.3, we might say, 'The sprinter accelerates **faster** than the car.' However, 'faster' really means 'greater speed'. It is better to say, 'The sprinter has a greater acceleration than the car.'

Acceleration is defined as follows:

acceleration = rate of change of velocity

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

So to calculate acceleration a, we need to know two quantities – the change in velocity Δv and the time taken Δt:

$$a = \frac{\Delta v}{\Delta t}$$

Sometimes this equation is written differently. We write u for the **initial velocity** and v for the **final velocity** (because u comes before v in the alphabet). The moving object accelerates from u to v in a time t (this is the same as the time represented by Δt above). Then the acceleration is given by the equation:

$$a = \frac{v - u}{t}$$

You must learn the definition of acceleration. It can be put in words or symbols. If you use symbols you must state what those symbols mean.

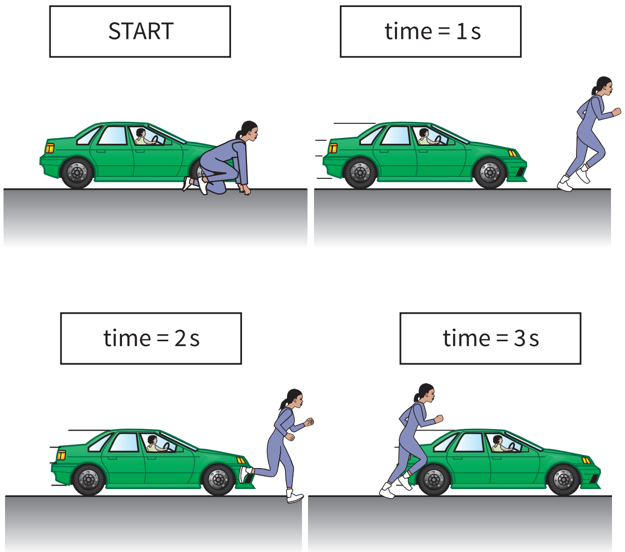


Figure 2.3 The sprinter has a greater acceleration than the car, but her top speed is less.

Units of acceleration

The unit of acceleration is $m\,s^{-2}$ (metres per second squared). The sprinter might have an acceleration of $5\,m\,s^{-2}$; her velocity increases by $5\,m\,s^{-1}$ every second. You could express acceleration in other units. For example, an advertisement might claim that a car accelerates from 0 to 60 miles per hour (mph) in 10 s. Its acceleration would then be $6\,mph\,s^{-1}$ (6 miles per hour per second). However, mixing together hours and seconds is not a good idea, and so acceleration is almost always given in the standard SI unit of $m\,s^{-2}$.

QUESTIONS

1 A car accelerates from a standing start and reaches a velocity of $18\,m\,s^{-1}$ after 6.0 s. Calculate its acceleration.

2 A car driver brakes gently. Her car slows down from $23\,m\,s^{-1}$ to $11\,m\,s^{-1}$ in 20 s. Calculate the magnitude (size) of her deceleration. (Note that, because she is slowing down, her acceleration is negative.)

3 A stone is dropped from the top of a cliff. Its acceleration is $9.81\,m\,s^{-2}$. How fast is it moving:

a after 1 s?

b after 3 s?

WORKED EXAMPLES

1 Leaving a bus stop, a bus reaches a velocity of $8.0\,m\,s^{-1}$ after 10 s. Calculate the acceleration of the bus.

Step 1 Note that the bus's initial velocity is $0\,m\,s^{-1}$. Therefore:

change in velocity $\Delta v = (8.0 - 0)\,m\,s^{-1}$

time taken $\Delta t = 10\,s$

Step 2 Substitute these values in the equation for acceleration:

$$\text{acceleration} = \frac{\Delta v}{\Delta t} = \frac{8.0}{10}$$

$$= 0.80\,m\,s^{-2}$$

2 A sprinter starting from rest has an acceleration of $5.0\,m\,s^{-2}$ during the first 2.0 s of a race. Calculate her velocity after 2.0 s.

Step 1 Rearranging the equation $a = \frac{v - u}{t}$ gives:

$v = u + at$

Step 2 Substituting the values and calculating gives:

$v = 0 + (5.0 \times 2.0) = 10\,m\,s^{-1}$

3 A train slows down from $60\,m\,s^{-1}$ to $20\,m\,s^{-1}$ in 50 s. Calculate the magnitude of the deceleration of the train.

Step 1 Write what you know:

$u = 60\,m\,s^{-1}$ $\quad v = 20\,m\,s^{-1}$ $\quad t = 50\,s$

Step 2 Take care! Here the train's final velocity is less than its initial velocity. To ensure that we arrive at the correct answer, we will use the alternative form of the equation to calculate a.

$$a = \frac{v - u}{t}$$

$$= \frac{20 - 60}{50} = \frac{-40}{50} = -0.80\,m\,s^{-2}$$

The minus sign (negative acceleration) indicates that the train is slowing down. It is decelerating. The magnitude of the deceleration is $0.80\,m\,s^{-2}$.

Deducing acceleration

The gradient of a velocity–time graph tells us whether the object's velocity has been changing at a high rate or a low rate, or not at all (Figure 2.4). We can deduce the value of the acceleration from the gradient of the graph:

acceleration = gradient of velocity–time graph

The graph (Figure 2.5) shows how the velocity of a cyclist changed during the start of a sprint race. We can find his acceleration during the first section of the graph (where the line is straight) using the triangle as shown.

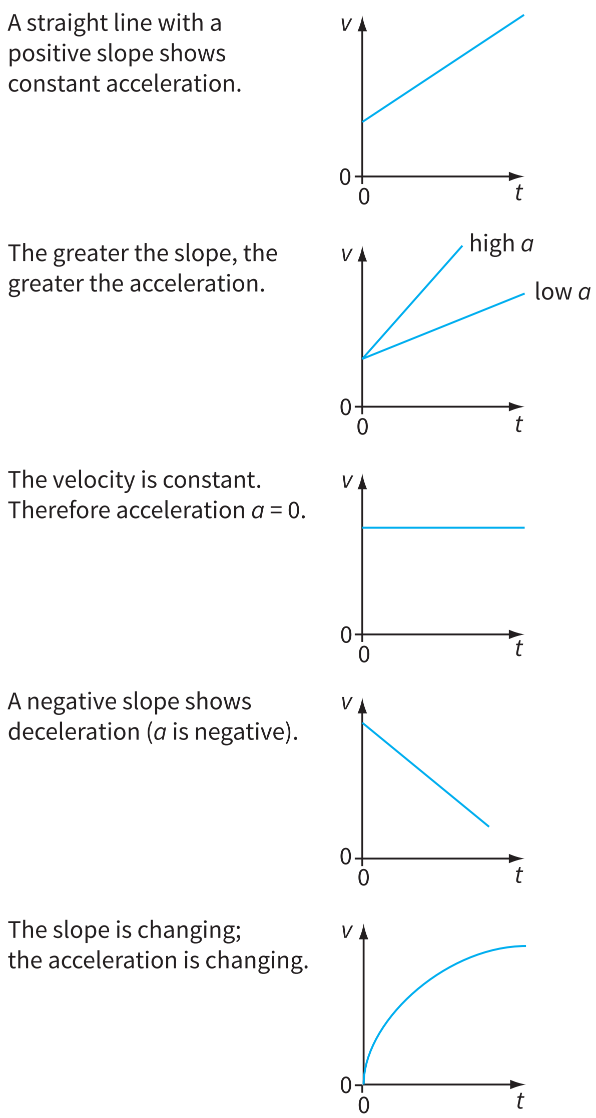


Figure 2.4 The gradient of a velocity–time graph is equal to acceleration.

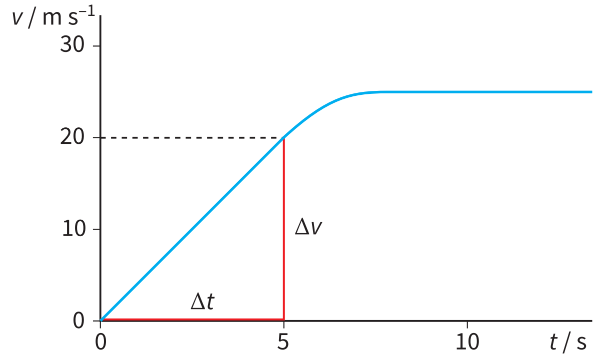


Figure 2.5 Deducing acceleration from a velocity–time graph.

The change in velocity Δv is given by the vertical side of the triangle. The time taken Δt is given by the horizontal side.

$$\text{acceleration} = \frac{\text{change in displacement}}{\text{time taken}}$$

$$= \frac{20-0}{5}$$

$$= 4.0\,\text{m s}^{-2}$$

A more complex example where the velocity–time graph is curved is shown on page 24.

Deducing displacement

We can also find the displacement of a moving object from its velocity–time graph. This is given by the area under the graph:

displacement = area under velocity–time graph

It is easy to see why this is the case for an object moving at a constant velocity. The displacement is simply velocity × time, which is the area of the shaded rectangle (Figure 2.6a).

For changing velocity, again the area under the graph gives displacement (Figure 2.6b). The area of each square of the graph represents a distance travelled: in this case, $1\,\text{m s}^{-1} \times 1\,\text{s}$, or 1 m. So, for this simple case in which the area is a triangle, we have:

$$\text{displacement} = \tfrac{1}{2} \times \text{base} \times \text{height}$$

$$= \tfrac{1}{2} \times 5.0 \times 10 = 25\,\text{m}$$

It is easy to confuse displacement–time graphs and velocity–time graphs. Check by looking at the quantity marked on the vertical axis.

For more complex graphs, you may have to use other techniques such as counting squares to deduce the area, but this is still equal to the displacement.

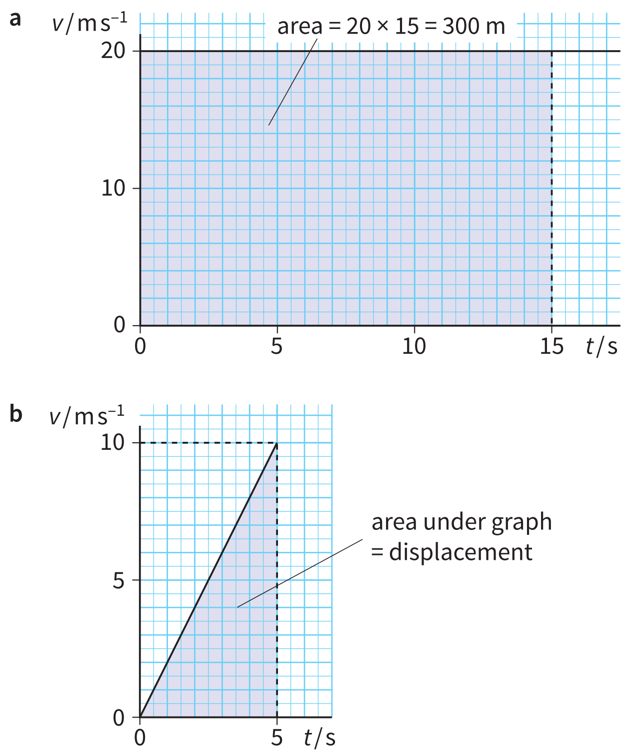


Figure 2.6 The area under the velocity–time graph is equal to the displacement of the object.

QUESTIONS

4 A lorry driver is travelling at the speed limit on a motorway. Ahead, he sees hazard lights and gradually slows down. He sees that an accident has occurred, and brakes suddenly to a halt. Sketch a velocity–time graph to represent the motion of this lorry.

5 Table 2.1 shows how the velocity of a motorcyclist changed during a speed trial along a straight road.
   - **a** Draw a velocity–time graph for this motion.
   - **b** From the table, deduce the motorcyclist's acceleration during the first 10 s.
   - **c** Check your answer by finding the gradient of the graph during the first 10 s.
   - **d** Determine the motorcyclist's acceleration during the last 15 s.
   - **e** Use the graph to find the total distance travelled during the speed trial.

Velocity / m s^{-1}	0	15	30	30	20	10	0
Time / s	0	5	10	15	20	25	30

Table 2.1 Data for a motorcyclist.

(Take care when counting squares: it is easiest when the sides of the squares stand for one unit. Check the axes, as the sides may represent 2 units, or 5 units, or some other number.)

Measuring velocity and acceleration

In a car crash, the occupants of the car may undergo a very rapid deceleration. This can cause them serious injury, but can be avoided if an air-bag is inflated within a fraction of a second. Figure 2.7 shows the tiny accelerometer at the heart of the system, which detects large accelerations and decelerations.

The acceleration sensor consists of two rows of interlocking teeth. In the event of a crash, these move relative to one another, and this generates a voltage which triggers the release of the air-bag.

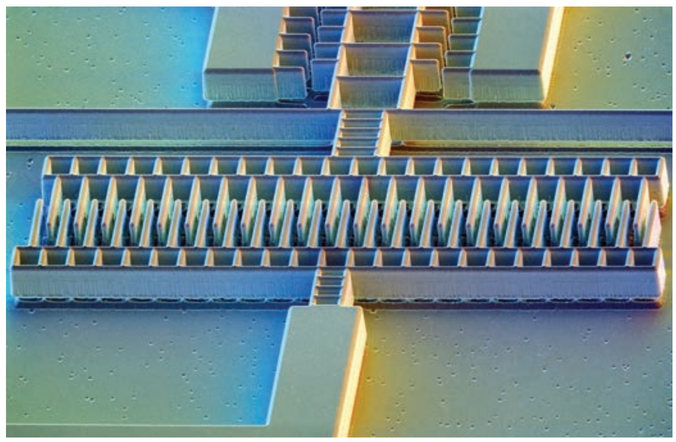


Figure 2.7 A micro-mechanical acceleration sensor is used to detect sudden accelerations and decelerations as a vehicle travels along the road. This electron microscope image shows the device magnified about 1000 times.

At the top of the photograph, you can see a second sensor which detects sideways accelerations. This is important in the case of a side impact.

These sensors can also be used to detect when a car swerves or skids, perhaps on an icy road. In this case, they activate the car's stability-control systems.

Determining velocity and acceleration in the laboratory

In Chapter 1, we looked at ways of finding the velocity of a trolley moving in a straight line. These involved measuring distance and time, and deducing velocity. Box 2.1 below shows how these techniques can be extended to find the acceleration of a trolley.

BOX 2.1: Laboratory measurements of acceleration

Measurements using light gates

The computer records the time for the first 'interrupt' section of the card to pass through the light beam of the light gate (Figure 2.8). Given the length of the interrupt, it can work out the trolley's initial velocity u. This is repeated for the second interrupt to give final velocity v. The computer also records the time interval $t_3 - t_1$ between these two velocity measurements. Now it can calculate the acceleration a as shown below:

$$u = \frac{l_1}{t_2 - t_1}$$

(l_1 = length of first section of the interrupt card)

and

$$v = \frac{l_2}{t_4 - t_3}$$

(l_2 = length of second section of the interrupt card)

Therefore:

$$a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$= \frac{v - u}{t_3 - t_1}$$

(Note that this calculation gives only an approximate value for a. This is because u and v are **average** speeds over a period of time; for an accurate answer we would need to know the speeds at times t_1 and t_3.)

Sometimes two light gates are used with a card of length l. The computer can still record the times as shown above and calculate the acceleration in the same way, with $l_1 = l_2 = l$.

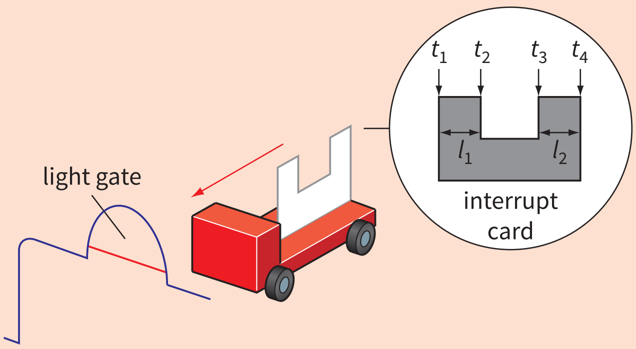


Figure 2.8 Determining acceleration using a single light gate.

Measurements using a ticker-timer

The practical arrangement is the same as for measuring velocity. Now we have to think about how to interpret the tape produced by an accelerating trolley (Figure 2.9).

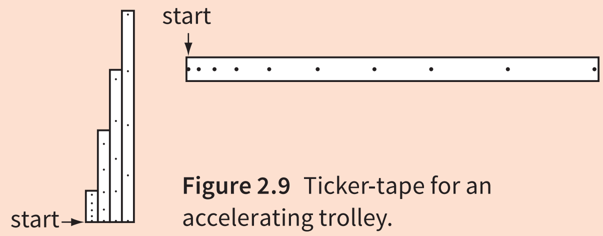


Figure 2.9 Ticker-tape for an accelerating trolley.

The tape is divided into sections, as before, every five dots. Remember that the time interval between adjacent dots is 0.02 s. Each section represents 0.10 s.

By placing the sections of tape side by side, you can picture the velocity–time graph.

The length of each section gives the trolley's displacement in 0.10 s, from which the average velocity during this time can be found. This can be repeated for each section of the tape, and a velocity–time graph drawn. The gradient of this graph is equal to the acceleration. Table 2.2 and Figure 2.10 show some typical results.

The acceleration is calculated to be:

$$a = \frac{\Delta v}{\Delta t} = \frac{0.93}{0.20} \approx 4.7\ \mathrm{m\,s^{-2}}$$

Section of tape	Time at start / s	Time interval / s	Length of section / cm	Velocity / m s^{-1}
1	0.0	0.10	2.3	0.23
2	0.10	0.10	7.0	0.70
3	0.20	0.10	11.6	1.16

Table 2.2 Data for Figure 2.10.

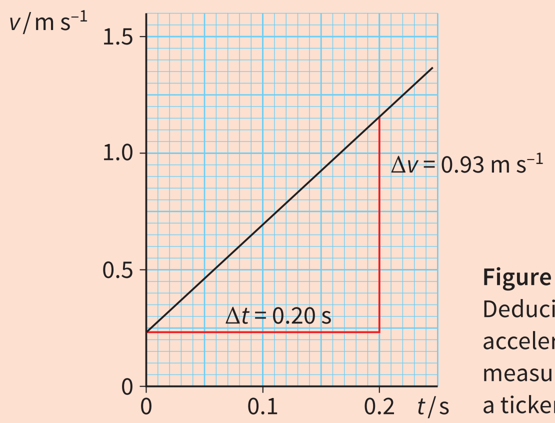


Figure 2.10 Deducing acceleration from measurements of a ticker-tape.

BOX 2.1: Laboratory measurements of acceleration (continued)

Measurements using a motion sensor

The computer software which handles the data provided by the motion sensor can calculate the acceleration of a trolley. However, because it deduces velocity from measurements of position, and then calculates acceleration from values of velocity, its precision is relatively poor.

QUESTIONS

6 Sketch a section of ticker-tape for a trolley which travels at a steady velocity and which then decelerates.

7 Figure 2.11 shows the dimensions of an interrupt card, together with the times recorded as it passed through a light gate. Use these measurements to calculate the acceleration of the card. (Follow the steps outlined on page 19.)

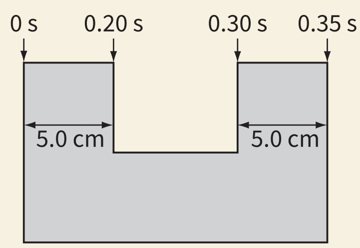


Figure 2.11 For Question 7.

8 Two adjacent five-dot sections of a ticker-tape measure 10 cm and 16 cm, respectively. The interval between dots is 0.02 s. Deduce the acceleration of the trolley which produced the tape.

The equations of motion

As a space rocket rises from the ground, its velocity steadily increases. It is accelerating (Figure 2.12).

Eventually it will reach a speed of several kilometres per second. Any astronauts aboard find themselves pushed back into their seats while the rocket is accelerating.

The engineers who planned the mission must be able to calculate how fast the rocket will be travelling and where it will be at any point in its journey. They have sophisticated computers to do this, using more elaborate versions of the equations given below.

Figure 2.12 A rocket accelerates as it lifts off from the ground.

There is a set of equations which allows us to calculate the quantities involved when an object is moving with a constant acceleration. The quantities we are concerned with are:

s	displacement	a	acceleration
u	initial velocity	t	time taken
v	final velocity		

Here are the four **equations of motion**.

equation 1: $v = u + at$

equation 2: $s = \frac{(u+v)}{2} \times t$

equation 3: $s = ut + \frac{1}{2}at^2$

equation 4: $v^2 = u^2 + 2as$

Take care using these equations. They can only be used:

- for motion in a straight line
- for an object with constant acceleration.

To get a feel for how to use these equations, we will consider some worked examples. In each example, we will follow the same procedure:

Step 1 We write down the quantities which we know, and the quantity we want to find.

Step 2 Then we choose the equation which links these quantities, and substitute in the values.

Step 3 Finally, we calculate the unknown quantity.

We will look at where these equations come from in the next section.

WORKED EXAMPLES

4 The rocket shown in Figure 2.12 lifts off from rest with an acceleration of $20\,\text{m s}^{-2}$. Calculate its velocity after 50 s.

Step 1 What we know: $u = 0\,\text{m s}^{-1}$, $a = 20\,\text{m s}^{-2}$, $t = 50\,\text{s}$

and what we want to know: $v = ?$

Step 2 The equation linking *u*, *a*, *t* and *v* is equation 1:

$v = u + at$

Substituting gives:

$v = 0 + (20 \times 50)$

Step 3 Calculation then gives:

$v = 1000\,\text{m s}^{-1}$

So the rocket will be travelling at $1000\,\text{m s}^{-1}$ after 50 s. This makes sense, since its velocity increases by $20\,\text{m s}^{-1}$ every second, for 50 s.

You could use the same equation to work out how long the rocket would take to reach a velocity of $2000\,\text{m s}^{-1}$, or the acceleration it must have to reach a speed of $1000\,\text{m s}^{-1}$ in 40 s, and so on.

5 The car shown in Figure 2.13 is travelling along a straight road at $8.0\,\text{m s}^{-1}$. It accelerates at $1.0\,\text{m s}^{-2}$ for a distance of 18 m. How fast is it then travelling?

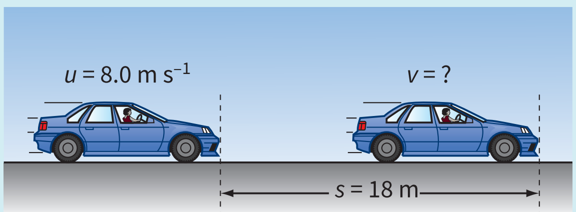


Figure 2.13 For Worked example 5. This car accelerates for a short distance as it travels along the road.

In this case, we will have to use a different equation, because we know the distance during which the car accelerates, not the time.

Step 1 What we know: $u = 8.0\,\text{m s}^{-1}$, $a = 1.0\,\text{m s}^{-2}$, $s = 18\,\text{m}$

and what we want to know: $v = ?$

Step 2 The equation we need is equation 4:

$v^2 = u^2 + 2as$

Substituting gives:

$v^2 = 8.0^2 + (2 \times 1.0 \times 18)$

Step 3 Calculation then gives:

$v^2 = 64 + 36 = 100\,\text{m}^2\,\text{s}^{-2}$

$v = 10\,\text{m s}^{-1}$

So the car will be travelling at $10\,\text{m s}^{-1}$ when it stops accelerating.

(You may find it easier to carry out these calculations without including the units of quantities when you substitute in the equation. However, including the units can help to ensure that you end up with the correct units for the final answer.)

6 A train (Figure 2.14) travelling at $20\,\text{m s}^{-1}$ accelerates at $0.50\,\text{m s}^{-2}$ for 30 s. Calculate the distance travelled by the train in this time.

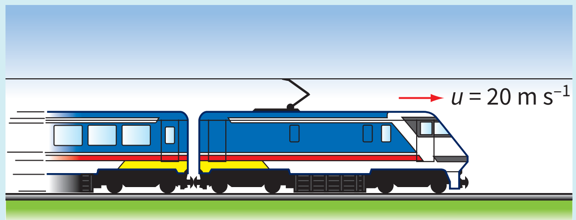


Figure 2.14 For Worked example 6. This train accelerates for 30 s.

Step 1 What we know: $u = 20\,\text{m s}^{-1}$, $t = 30\,\text{s}$, $a = 0.50\,\text{m s}^{-2}$

and what we want to know: $s = ?$

Step 2 The equation we need is equation 3:

$s = ut + \frac{1}{2}at^2$

Substituting gives:

$s = (20 \times 30) + \frac{1}{2} \times 0.5 \times (30)^2$

Step 3 Calculation then gives:

$s = 600 + 225 = 825\,\text{m}$

So the train will travel 825 m while it is accelerating.

WORKED EXAMPLES (continued)

7 The cyclist in Figure 2.15 is travelling at $15\,\mathrm{m\,s^{-1}}$. She brakes so that she doesn't collide with the wall. Calculate the magnitude of her deceleration.

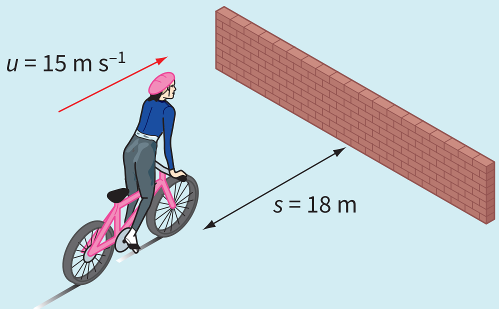


Figure 2.15 For Worked example 7. The cyclist brakes to stop herself colliding with the wall.

This example shows that it is sometimes necessary to rearrange an equation, to make the unknown quantity its subject. It is easiest to do this before substituting in the values.

Step 1 What we know: $u = 15\,\mathrm{m\,s^{-1}}$
$v = 0\,\mathrm{m\,s^{-1}}$
$s = 18\,\mathrm{m}$

and what we want to know: $a = ?$

Step 2 The equation we need is equation 4:

$$v^2 = u^2 + 2as$$

Rearranging gives:

$$a = \frac{v^2 - u^2}{2s}$$

$$a = \frac{0^2 - 15^2}{2 \times 18} = \frac{-225}{36}$$

Step 3 Calculation then gives:

$$a = -6.25\,\mathrm{m\,s^{-2}} \approx -6.3\,\mathrm{m\,s^{-2}}$$

So the cyclist will have to brake hard to achieve a deceleration of magnitude $6.3\,\mathrm{m\,s^{-2}}$. The minus sign shows that her acceleration is negative, i.e. a deceleration.

QUESTIONS

9 A car is initially stationary. It has a constant acceleration of $2.0\,\mathrm{m\,s^{-2}}$.

- **a** Calculate the velocity of the car after 10 s.
- **b** Calculate the distance travelled by the car at the end of 10 s.
- **c** Calculate the time taken by the car to reach a velocity of $24\,\mathrm{m\,s^{-1}}$.

10 A train accelerates steadily from $4.0\,\mathrm{m\,s^{-1}}$ to $20\,\mathrm{m\,s^{-1}}$ in 100 s.

- **a** Calculate the acceleration of the train.
- **b** From its initial and final velocities, calculate the average velocity of the train.
- **c** Calculate the distance travelled by the train in this time of 100 s.

11 A car is moving at $8.0\,\mathrm{m\,s^{-1}}$. The driver makes it accelerate at $1.0\,\mathrm{m\,s^{-2}}$ for a distance of 18 m. What is the final velocity of the car?

Deriving the equations of motion

On the previous pages, we have seen how to make use of the equations of motion. But where do these equations come from? They arise from the definitions of velocity and acceleration.

We can find the first two equations from the velocity–time graph shown in Figure 2.16. The graph represents the motion of an object. Its initial velocity is u. After time t, its final velocity is v.

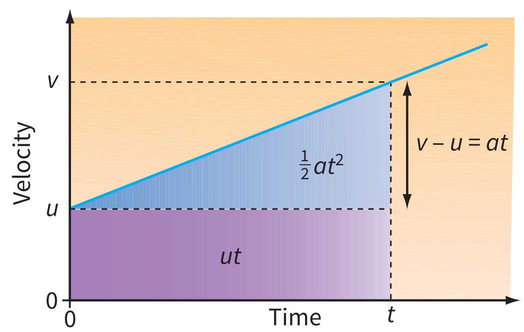


Figure 2.16 This graph shows the variation of velocity of an object with time. The object has constant acceleration.

Equation 1

The graph of Figure 2.16 is a straight line, therefore the object's acceleration a is constant. The gradient (slope) of the line is equal to acceleration.

The acceleration is defined as:

$$a = \frac{(v-u)}{t}$$

which is the gradient of the line. Rearranging this gives the first equation of motion:

$$v = u + at \qquad \text{(equation 1)}$$

Equation 2

Displacement is given by the area under the velocity–time graph. Figure 2.17 shows that the object's average velocity is half-way between u and v. So the object's average velocity, calculated by averaging its initial and final velocities, is given by:

$$\frac{(u-v)}{2}$$

The object's displacement is the shaded area in Figure 2.17. This is a rectangle, and so we have:

displacement = average velocity × time taken

and hence:

$$s = \frac{(u+v)}{2} \times t \qquad \text{(equation 2)}$$

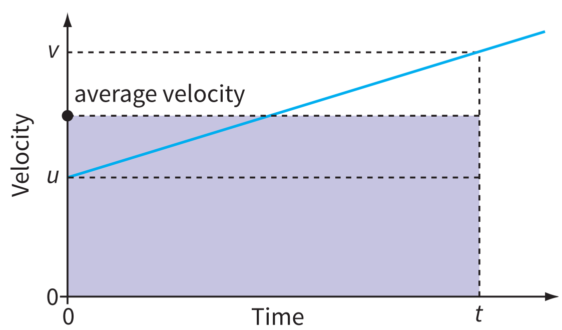


Figure 2.17 The average velocity is half-way between u and v.

Equation 3

From equations 1 and 2, we can derive equation 3:

$$v = u + at \qquad \text{(equation 1)}$$

$$s = \frac{(u+v)}{2} \times t \qquad \text{(equation 2)}$$

Substituting v from equation 1 gives:

$$s = \frac{(u+u+at)}{2} \times t$$

$$s = \frac{2ut}{2} + \frac{at^2}{2}$$

So

$$s = ut + \tfrac{1}{2}at^2 \qquad \text{(equation 3)}$$

Looking at Figure 2.16, you can see that the two terms on the right of the equation correspond to the areas of the rectangle and the triangle which make up the area under the graph. Of course, this is the same area as the rectangle in Figure 2.17.

Equation 4

Equation 4 is also derived from equations 1 and 2:

$$v = u + at \qquad \text{(equation 1)}$$

$$s = \frac{(u+u)}{2} \times t \qquad \text{(equation 2)}$$

Substituting for time t from equation 1 gives:

$$s = \frac{(u+v)}{2} + \frac{(v+u)}{a} \qquad \text{(equation 2)}$$

Rearranging this gives:

$$2as = (u+v)(v-u)$$

$$= v^2 - u^2$$

or simply:

$$v^2 = u^2 + 2as \qquad \text{(equation 4)}$$

Investigating road traffic accidents

The police frequently have to investigate road traffic accidents. They make use of many aspects of physics, including the equations of motion. The next two questions will help you to apply what you have learned to situations where police investigators have used evidence from skid marks on the road.

QUESTIONS

12 Trials on the surface of a new road show that, when a car skids to a halt, its acceleration is $-7.0\,\text{m s}^{-2}$. Estimate the skid-to-stop distance of a car travelling at a speed limit of $30\,\text{m s}^{-1}$ (approx. $110\,\text{km h}^{-1}$ or 70 mph).

13 At the scene of an accident on a country road, police find skid marks stretching for 50 m. Tests on the road surface show that a skidding car decelerates at $6.5\,\text{m s}^{-2}$. Was the car which skidded exceeding the speed limit of $25\,\text{m s}^{-1}$ ($90\,\text{km h}^{-1}$) on this road?

Uniform and non-uniform acceleration

It is important to note that the equations of motion only apply to an object which is moving with a constant acceleration. If the acceleration *a* was changing, you wouldn't know what value to put in the equations. Constant acceleration is often referred to as uniform acceleration.

The velocity–time graph in Figure 2.18 shows **non-uniform** acceleration. It is not a straight line; its gradient is changing (in this case, decreasing).

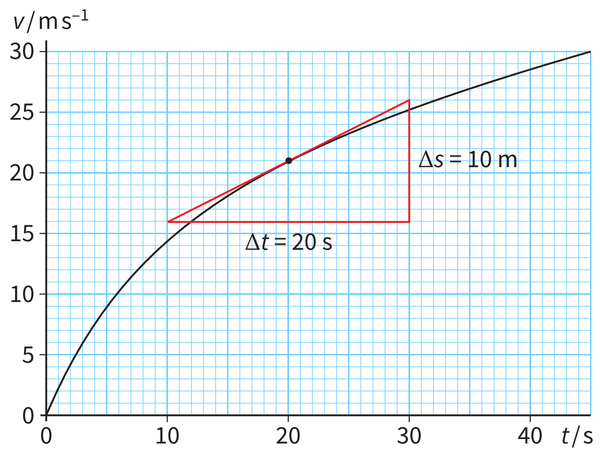


Figure 2.18 This curved velocity–time graph cannot be analysed using the equations of motion.

The acceleration at any instant in time is given by the gradient of the velocity–time graph. The triangle in Figure 2.18 shows how to find the acceleration at t = 20 seconds:

- At the time of interest, mark a point on the graph.
- Draw a **tangent** to the curve at that point.
- Make a large right-angled triangle, and use it to find the gradient.

You can find the change in displacement of the body as it accelerates by determining the area under the velocity–time graph.

To find the displacement of the object in Figure 2.18 between t = 0 and 20 s, the most straightforward, but lengthy, method is just to count the number of small squares.

In this case up to t = 20 s, there are about 250 small squares. This is tedious to count but you can save yourself a lot of time by drawing a line from the origin to the point at 20 s. The area of the triangle is easy to find (200 small squares) and then you only have to count the number of small squares between the line you have drawn and the curve on the graph (about 50 squares)

In this case each square is 1 m s^{-1} on the y-axis by 1 s on the x-axis, so the area of each square is 1 × 1 = 1 m and the displacement is 250 m. In other cases note carefully the value of each side of the square you have chosen.

QUESTIONS

14 The graph in Figure 2.19 represents the motion of an object moving with varying acceleration. Lay your ruler on the diagram so that it is tangential to the graph at point P.

a What are the values of time and velocity at this point?

b Estimate the object's acceleration at this point.

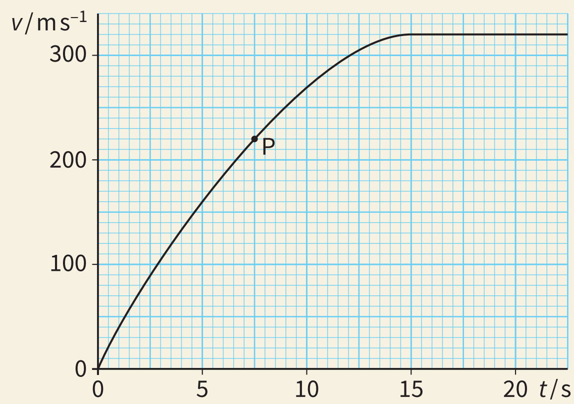


Figure 2.19 For Question 14.

15 The velocity–time graph (Figure 2.20) represents the motion of a car along a straight road for a period of 30 s.

a Describe the motion of the car.

b From the graph, determine the car's initial and final velocities over the time of 30 s.

c Determine the acceleration of the car.

d By calculating the area under the graph, determine the displacement of the car.

e Check your answer to part **d** by calculating the car's displacement using $s = ut + \frac{1}{2}at^2$.

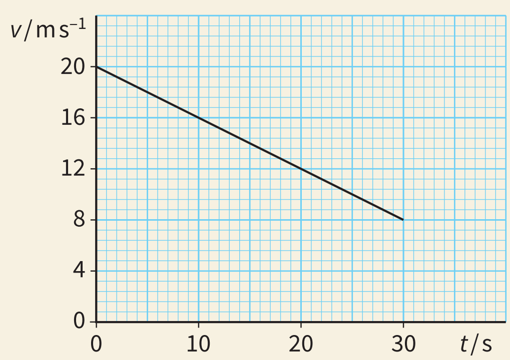


Figure 2.20 For Question 15.

Acceleration caused by gravity

If you drop a ball or stone, it falls to the ground. Figure 2.21, based on a multiflash photograph, shows the ball at equal intervals of time. You can see that the ball's velocity increases as it falls because the spaces between the images of the ball increase steadily. The ball is accelerating.

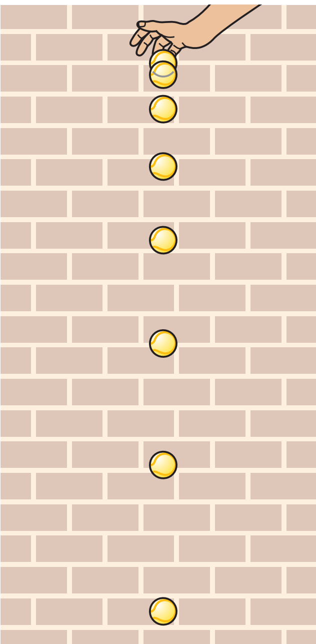


Figure 2.21 This diagram of a falling ball, based on a multiflash photo, clearly shows that the ball's velocity increases as it falls.

A multiflash photograph is useful to demonstrate that the ball accelerates as it falls. Usually, objects fall too quickly for our eyes to be able to observe them speeding up. It is easy to imagine that the ball moves quickly as soon as you let it go, and falls at a steady speed to the ground. Figure 2.21 shows that this is not the case.

If we measure the acceleration of a freely falling object on the surface of the Earth, we find a value of about $9.81\ \text{m s}^{-2}$. This is known as the **acceleration of free fall**, and is given the symbol g:

acceleration of free fall, $g = 9.81\ \text{m s}^{-2}$

The value of g depends on where you are on the Earth's surface, but we usually take $g = 9.81\ \text{m s}^{-2}$.

If we drop an object, its initial velocity $u = 0$. How far will it fall in time t? Substituting in $s = ut + \frac{1}{2}at^2$ gives displacement s:

$$s = \tfrac{1}{2} \times 9.81 \times t^2$$

$$= 4.9 \times t^2$$

Hence, by timing a falling object, we can determine g.

QUESTIONS

16 If you drop a stone from the edge of a cliff, its initial velocity $u = 0$, and it falls with acceleration $g = 9.81\ \text{m s}^{-2}$. You can calculate the distance s it falls in a given time t using an equation of motion.

- **a** Copy and complete Table 2.3, which shows how s depends on t.
- **b** Draw a graph of s against t.
- **c** Use your graph to find the distance fallen by the stone in 2.5 s.
- **d** Use your graph to find how long it will take the stone to fall to the bottom of a cliff 40 m high. Check your answer using the equations of motion.

Time / s	0	1.0	2.0	3.0	4.0
Displacement / m	0	4.9			

Table 2.3 Time (t) and displacement (s) data for Question 16.

17 An egg falls off a table. The floor is 0.8 m from the table-top.

- **a** Calculate the time taken to reach the ground.
- **b** Calculate the velocity of impact with the ground.

Determining *g*

One way to measure the acceleration of free fall g would be to try bungee-jumping (Figure 2.22). You would need to carry a stopwatch, and measure the time between jumping from the platform and the moment when the elastic rope begins to slow your fall. If you knew the length of the unstretched rope, you could calculate g.

There are easier methods for finding g which can be used in the laboratory. These are described in Box 2.2.

Figure 2.22 A bungee-jumper falls with initial acceleration g.

BOX 2.2: Laboratory measurements of *g*

Measuring *g* using an electronic timer

In this method, a steel ball-bearing is held by an electromagnet (Figure 2.23). When the current to the magnet is switched off, the ball begins to fall and an electronic timer starts. The ball falls through a trapdoor, and this breaks a circuit to stop the timer. This tells us the time taken for the ball to fall from rest through the distance *h* between the bottom of the ball and the trapdoor.

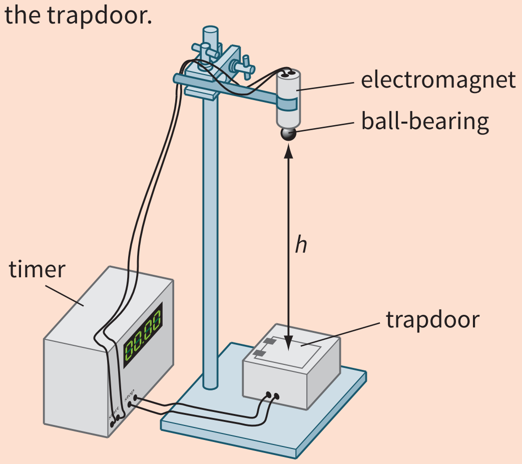


Figure 2.23 The timer records the time for the ball to fall through the distance *h*.

Here is how we can use one of the equations of motion to find *g*:

displacement $s = h$
time taken $= t$
initial velocity $u = 0$
acceleration $a = g$

Substituting in $s = ut + \frac{1}{2}at^2$ gives:

$$h = \frac{1}{2}gt^2$$

and for any values of *h* and *t* we can calculate a value for *g*.

A more satisfactory procedure is to take measurements of *t* for several different values of *h*. The height of the ball bearing above the trapdoor is varied systematically, and the time of fall measured several times to calculate an average for each height. Table 2.4 and Figure 2.24 show some typical results. We can deduce g from the gradient of the graph of *h* against t^2. The equation for a straight line through the origin is:

$$y = mx$$

In our experiment we have:

$$h = \frac{1}{2}g \; t^2$$
$$y = m \; x$$

h / m	*t* / s	t^2 / s^2
0.27	0.25	0.063
0.39	0.30	0.090
0.56	0.36	0.130
0.70	0.41	0.168
0.90	0.46	0.212

Table 2.4 Data for Figure 2.24. These are mean values.

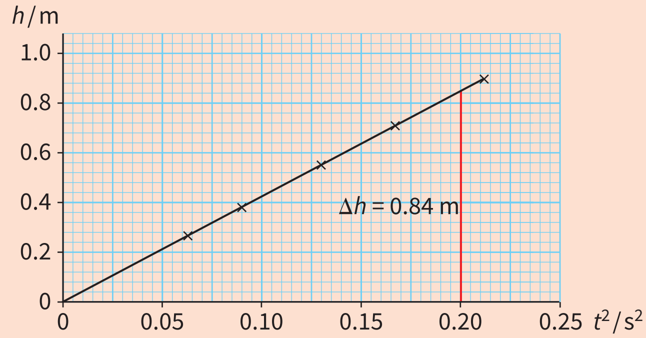


Figure 2.24 The acceleration of free fall can be determined from the gradient.

The gradient of the straight line of a graph of *h* against t^2 is equal to $\frac{g}{2}$. Therefore:

$$\text{gradient} = \frac{g}{2} = \frac{0.84}{0.20} = 4.2$$

$$g = 4.2 \times 2 = 8.4\,\text{m s}^{-2}$$

Sources of uncertainty

The electromagnet may retain some magnetism when it is switched off, and this may tend to slow the ball's fall. Consequently, the time *t* recorded by the timer may be longer than if the ball were to fall completely freely. From $h = \frac{1}{2}gt^2$, it follows that, if *t* is too great, the experimental value of *g* will be too small. This is an example of a systematic error – all the results are systematically distorted so that they are too great (or too small) as a consequence of the experimental design.

Measuring the height *h* is awkward. You can probably only find the value of *h* to within ±1 mm at best. So there is a random error in the value of *h*, and this will result in a slight scatter of the points on the graph, and a degree of uncertainty in the final value of *g*. For more about errors, see P1: Practical skills for AS.

BOX 2.2: Laboratory measurements of *g* (continued)

Measuring *g* using a ticker-timer

Figure 2.25 shows a weight falling. As it falls, it pulls a tape through a ticker-timer. The spacing of the dots on the tape increases steadily, showing that the weight is accelerating. You can analyse the tape to find the acceleration, as discussed on page 19.

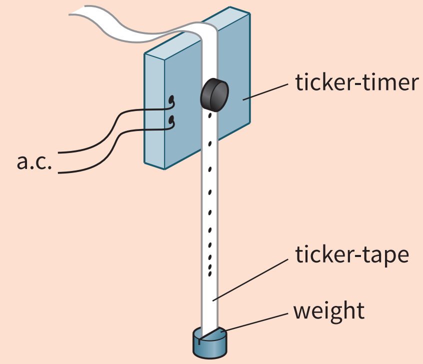


Figure 2.25 A falling weight pulls a tape through a ticker-timer.

This is not a very satisfactory method of measuring *g*. The main problem arises from friction between the tape and the ticker-timer. This slows the fall of the weight and so its acceleration is less than *g*. (This is another example of a systematic error.)

The effect of friction is less of a problem for a large weight, which falls more freely. If measurements are made for increasing weights, the value of acceleration gets closer and closer to the true value of *g*.

Measuring *g* using a light gate

Figure 2.26 shows how a weight can be attached to a card 'interrupt'. The card is designed to break the light beam twice as the weight falls. The computer can then calculate the velocity of the weight twice as it falls, and hence find its acceleration:

$$\text{initial velocity } u = \frac{x}{t_2 - t_1}$$

$$\text{final velocity } v = \frac{x}{t_4 - t_3}$$

Therefore:

$$\text{acceleration } a = \frac{v - u}{t_3 - t_1}$$

The weight can be dropped from different heights above the light gate. This allows you to find out whether its acceleration is the same at different points in its fall. This is an advantage over Method 1, which can only measure the acceleration from a stationary start.

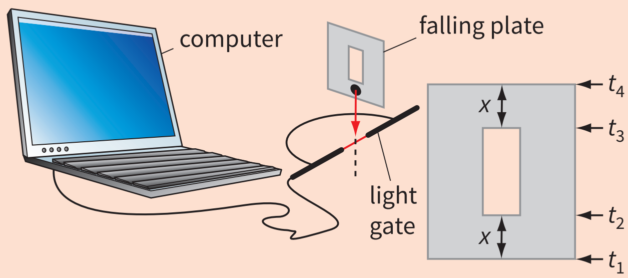


Figure 2.26 The weight accelerates as it falls. The upper section of the card falls more quickly through the light gate.

WORKED EXAMPLE

8 To get a rough value for *g*, a student dropped a stone from the top of a cliff. A second student timed the stone's fall using a stopwatch. Here are their results:

estimated height of cliff = 30 m

time of fall = 2.6 s

Use the results to estimate a value for *g*.

Step 1 Calculate the average speed of the stone:

$$\text{average speed of stone during fall} = \frac{30}{2.6} = 11.5\,\text{m s}^{-1}$$

Step 2 Find the values of *v* and *u*:

final speed $v = 2 \times 11.5\,\text{m s}^{-1} = 23.0\,\text{m s}^{-1}$

initial speed $u = 0\,\text{m s}^{-1}$

Step 3 Substitute these values into the equation for acceleration:

$$a = \frac{v - u}{t} = \frac{23.0}{2.6} = 8.8\,\text{m s}^{-2}$$

Note that you can reach the same result more directly using $s = ut + \frac{1}{2}at^2$, but you may find it easier to follow what is going on using the method given here. We should briefly consider why the answer is less than the expected value of $g = 9.81\,\text{m s}^{-2}$. It might be that the cliff was higher than the student's estimate. The timer may not have been accurate in switching the stopwatch on and off. There will have been air resistance which slowed the stone's fall.

QUESTIONS

18 A steel ball falls from rest through a height of 2.10 m. An electronic timer records a time of 0.67 s for the fall.

a Calculate the average acceleration of the ball as it falls.

b Suggest reasons why the answer is not exactly $9.81\,m\,s^{-2}$.

19 In an experiment to determine the acceleration due to gravity, a ball was timed electronically as it fell from rest through a height *h*. The times *t* shown in Table 2.5 were obtained.

a Plot a graph of *h* against t^2.

b From the graph, determine the acceleration of free fall, *g*.

c Comment on your answer.

Height / m	0.70	1.03	1.25	1.60	1.99
Time / s	0.99	1.13	1.28	1.42	1.60

Table 2.5 Height (*h*) and time (*t*) data for Question 19.

20 In Chapter 1, we looked at how to use a motion sensor to measure the speed and position of a moving object. Suggest how a motion sensor could be used to determine *g*.

Motion in two dimensions – projectiles

A curved trajectory

A multiflash photograph can reveal details of the path, or trajectory, of a projectile. Figure 2.27 shows the trajectories of a projectile – a bouncing ball. Once the ball has left the child's hand and is moving through the air, the only force acting on it is its weight.

The ball has been thrown at an angle to the horizontal. It speeds up as it falls – you can see that the images of the ball become further and further apart. At the same time, it moves steadily to the right. You can see this from the even spacing of the images across the picture. The ball's path has a mathematical shape known as a **parabola**. After it bounces, the ball is moving more slowly. It slows down, or decelerates, as it rises – the images get closer and closer together.

We interpret this picture as follows. The vertical motion of the ball is affected by the force of gravity, that is, its weight. When it rises it has a vertical deceleration

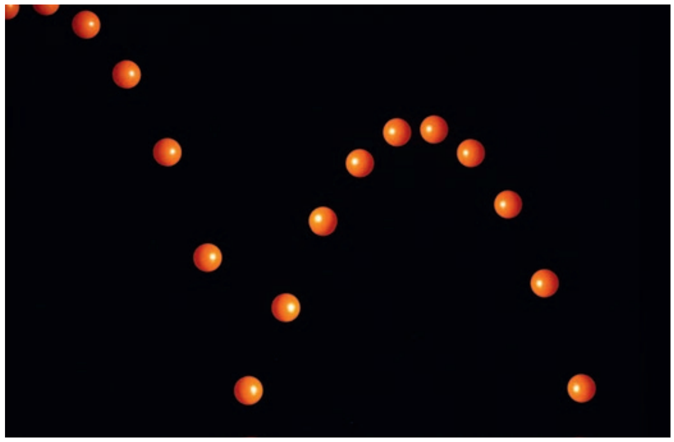


Figure 2.27 A bouncing ball is an example of a projectile. This multiflash photograph shows details of its motion which would escape the eye of an observer.

of magnitude *g*, which slows it down, and when it falls it has an acceleration of *g*, which speeds it up. The ball's horizontal motion is unaffected by gravity. In the absence of air resistance, the ball has a constant velocity in the horizontal direction. We can treat the ball's vertical and horizontal motions separately, because they are independent of one another.

Components of a vector

In order to understand how to treat the velocity in the vertical and horizontal directions separately we start by considering a constant velocity.

If an aeroplane has a constant velocity *v* at an angle θ as shown in Figure 2.28, then we say that this velocity has two effects or **components**, v_N in a northerly direction and v_E in an easterly direction. These two components of velocity add up to make the actual velocity *v*.

This process of taking a velocity and determining its effect along another direction is known as **resolving** the velocity along a different direction. In effect splitting the velocity into two components at right angles is the reverse

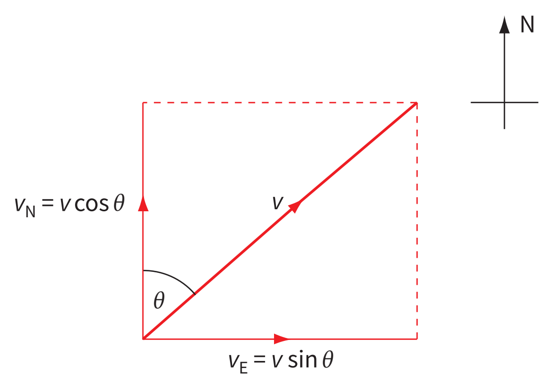


Figure 2.28 Components of a velocity. The component due north is $v_N = v\cos\theta$ and the component due east is $v_E = v\sin\theta$.

of adding together two vectors – it is splitting one vector into two vectors along convenient directions.

To find the component of any vector (e.g. displacement, velocity, acceleration) in a particular direction, we can use the following strategy:

Step 1 Find the angle θ between the vector and the direction of interest.

Step 2 Multiply the vector by the cosine of the angle θ.

So the component of an object's velocity v at angle θ to v is equal to $v\cos\theta$ (Figure 2.28).

QUESTION

21 Find the x- and y-components of each of the vectors shown in Figure 2.29. (You will need to use a protractor to measure angles from the diagram.)

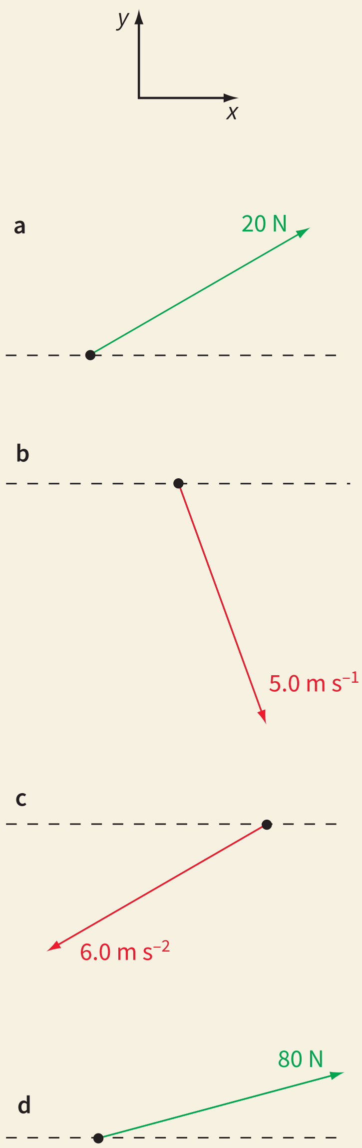


Figure 2.29 The vectors for Question 21.

Understanding projectiles

We will first consider the simple case of a projectile thrown straight up in the air, so that it moves vertically. Then we will look at projectiles which move horizontally and vertically at the same time.

Up and down

A stone is thrown upwards with an initial velocity of $20\,\text{m}\,\text{s}^{-1}$. Figure 2.30 shows the situation.

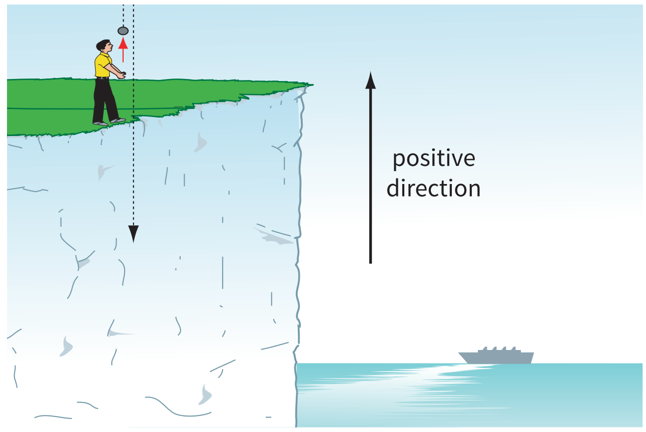


Figure 2.30 Standing at the edge of the cliff, you throw a stone vertically upwards. The height of the cliff is 25 m.

It is important to use a consistent sign convention here. We will take upwards as positive, and downwards as negative. So the stone's initial velocity is positive, but its acceleration g is negative. We can solve various problems about the stone's motion by using the equations of motion.

How high?

How high will the stone rise above ground level of the cliff?

As the stone rises upwards, it moves more and more slowly – it decelerates, because of the force of gravity. At its highest point, the stone's velocity is zero. So the quantities we know are:

initial velocity $= u = 20\,\text{m}\,\text{s}^{-1}$

final velocity $= v = 0\,\text{m}\,\text{s}^{-1}$

acceleration $= a = -9.81\,\text{m}\,\text{s}^{-2}$

displacement $= s = ?$

The relevant equation of motion is $v^2 = u^2 + 2as$. Substituting values gives:

$$0^2 = 20^2 + 2 \times (-9.81) \times s$$

$$0 = 400 - 19.62s$$

$$s = \frac{400}{19.62} = 20.4\,\text{m} \approx 20\,\text{m}$$

The stone rises 20 m upwards, before it starts to fall again.

How long?

How long will it take from leaving your hand for the stone to fall back to the clifftop?

When the stone returns to the point from which it was thrown, its displacement s is zero. So:

$s = 0 \quad u = 20\,\text{m s}^{-1} \quad a = -9.81\,\text{m s}^{-2} \quad t = ?$

Substituting in $s = ut + \frac{1}{2}at^2$ gives:

$$0 = 20t \times \tfrac{1}{2}(-9.81) \times t^2$$

$$= 20t - 4.905t^2 = (20 - 4.905t) \times t$$

There are two possible solutions to this:

- $t = 0$ s, i.e. the stone had zero displacement at the instant it was thrown
- $t = 4.1$ s, i.e. the stone returned to zero displacement after 4.1 s, which is the answer we are interested in.

Falling further

The height of the cliff is 25 m. How long will it take the stone to reach the foot of the cliff?

This is similar to the last example, but now the stone's final displacement is 25 m below its starting point. By our sign convention, this is a negative displacement, and $s = -25$ m.

QUESTIONS

22 In the example above (Falling further), calculate the time it will take for the stone to reach the foot of the cliff.

23 A ball is fired upwards with an initial velocity of $30\,\text{m s}^{-1}$. Table 2.6 shows how the ball's velocity changes. (Take $g = 9.81\,\text{m s}^{-2}$.)

a Copy and complete the table.

b Draw a graph to represent the data.

c Use your graph to deduce how long the ball took to reach its highest point.

Velocity / m s^{-1}	30	20.19				
Time / s	0	1.0	2.0	3.0	4.0	5.0

Table 2.6 For Question 23.

Vertical and horizontal at the same time

Here is an example to illustrate what happens when an object travels vertically and horizontally at the same time.

In a toy, a ball-bearing is fired horizontally from a point 0.4 m above the ground. Its initial velocity is $2.5\,\text{m s}^{-1}$. Its positions at equal intervals of time have been calculated and are shown in Table 2.7. These results are also shown in Figure 2.31. Study the table and the graph. You should notice the following:

- The horizontal distance increases steadily. This is because the ball's horizontal motion is unaffected by the force of gravity. It travels at a steady velocity horizontally so we can use $v = \frac{s}{t}$.
- The vertical distances do not show the same pattern. The ball is accelerating downwards so we must use the equations of motion. (These figures have been calculated using $g = 9.81\,\text{m s}^{-2}$.)

Time / s	Horizontal distance / m	Vertical distance / m
0.00	0.00	0.000
0.04	0.10	0.008
0.08	0.20	0.031
0.12	0.30	0.071
0.16	0.40	0.126
0.20	0.50	0.196
0.24	0.60	0.283
0.28	0.70	0.385

Table 2.7 Data for the example of a moving ball, as shown in Figure 2.31.

You can calculate the distance s fallen using the equation of motion $s = ut + \frac{1}{2}at^2$. (The initial vertical velocity $u = 0$.)

The horizontal distance is calculated using:

horizontal distance $= 2.5 \times t$

The vertical distance is calculated using:

vertical distance $= \frac{1}{2} \times 9.81 \times t^2$

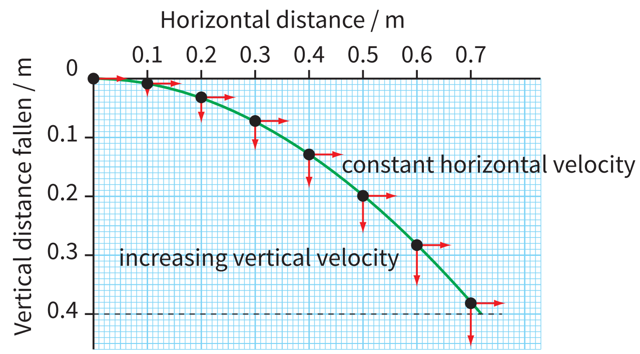


Figure 2.31 This sketch shows the path of the ball projected horizontally. The arrows represent the horizontal and vertical components of its velocity.

WORKED EXAMPLES

9 A stone is thrown horizontally with a velocity of 12 m s^{-1} from the top of a vertical cliff.

Calculate how long the stone takes to reach the ground 40 m below and how far the stone lands from the base of the cliff.

Step 1 Consider the ball's vertical motion. It has zero initial speed vertically and travels 40 m with acceleration 9.81 m s^{-2} in the same direction.

$$s = ut + \frac{1}{2}at^2$$

$$40 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

Thus t = 2.86 s.

Step 2 Consider the ball's horizontal motion. The ball travels with a constant horizontal velocity, 12 m s^{-1}, as long as there is no air resistance.

distance travelled = $u \times t$ = 12 × 2.86 = 34.3 m

Hint: *You may find it easier to summarise the information like this:*

vertically	$s = 40$	$u = 0$	$a = 9.81$	$t = ?$	$v = ?$
horizontally	$u = 12$	$v = 12$	$a = 0$	$t = ?$	$s = ?$

10 A ball is thrown with an initial velocity of 20 m s^{-1} at an angle of 30° to the horizontal (Figure 2.32). Calculate the horizontal distance travelled by the ball (its **range**).

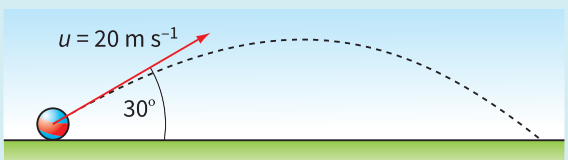


Figure 2.32 Where will the ball land?

Step 1 Split the ball's initial velocity into horizontal and vertical components:

initial velocity = u = 20 m s^{-1}

horizontal component of initial velocity

$= u\cos\theta = 20 \times \cos 30° = 17.3$ m s^{-1}

vertical component of initial velocity

$= u\sin\theta = 20 \times \sin 30° = 10$ m s^{-1}

Step 2 Consider the ball's vertical motion. How long will it take to return to the ground? In other words, when will its displacement return to zero?

$u = 10$ m s^{-1} $a = -9.81$ m s^{-2} $s = 0$ $t = ?$

Using $s = ut + \frac{1}{2}at^2$, we have:

$0 = 10t - 4.905t^2$

This gives t = 0 s or t = 2.04 s. So the ball is in the air for 2.04 s.

Step 3 Consider the ball's horizontal motion. How far will it travel horizontally in the 2.04 s before it lands? This is simple to calculate, since it moves with a constant horizontal velocity of 17.3 m s^{-1}.

horizontal displacement s = 17.3 × 2.04

= 35.3 m

Hence the horizontal distance travelled by the ball (its range) is about 35 m.

QUESTIONS

24 A stone is thrown horizontally from the top of a vertical cliff and lands 4.0 s later at a distance 12.0 m from the base of the cliff. Ignore air resistance.

a Calculate the horizontal speed of the stone.

b Calculate the height of the cliff.

25 A stone is thrown with a velocity of 8 m s^{-1} into the air at an angle of 40° to the horizontal.

a Calculate the vertical component of the velocity.

b State the value of the vertical component of the velocity when the stone reaches its highest point. Ignore air resistance.

c Use your answers to **a** and **b** to calculate the time the stone takes to reach it highest point.

d Calculate the horizontal component of the velocity.

e Use your answers to **c** and **d** to find the horizontal distance travelled by the stone as it climbs to its highest point.

26 The range of a projectile is the horizontal distance it travels before it reaches the ground. The greatest range is achieved if the projectile is thrown at 45° to the horizontal.

A ball is thrown with an initial velocity of 40 m s^{-1}. Calculate its greatest possible range when air resistance is considered to be negligible.

Summary

- Acceleration is equal to the rate of change of velocity.
- Acceleration is a vector quantity.
- The gradient of a velocity–time graph is equal to acceleration:

  $a = \frac{\Delta v}{\Delta t}$

- The area under a velocity–time graph is equal to displacement (or distance travelled).
- The equations of motion (for constant acceleration in a straight line) are:

  $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$

  $s = \frac{(u + v)}{2}$ $\quad v^2 = u^2 + 2as$

- Vectors such as forces can be resolved into components. Components at right angles to one another can be treated independently of one another. For a velocity v at an angle θ to the x-direction, the components are:

  x-direction: $v\cos\theta$

  y-direction: $v\sin\theta$

- For projectiles, the horizontal and vertical components of velocity can be treated independently. In the absence of air resistance, the horizontal component of velocity is constant while the vertical component of velocity downwards increases at a rate of $9.81\,\mathrm{m\,s^{-2}}$.

End-of-chapter questions

1 A motorway designer can assume that cars approaching a motorway enter a slip road with a velocity of $10\,\mathrm{m\,s^{-1}}$ and reach a velocity of $30\,\mathrm{m\,s^{-1}}$ before joining the motorway. Calculate the minimum length for the slip road, assuming that vehicles have an acceleration of $4.0\,\mathrm{m\,s^{-2}}$. [4]

2 A train is travelling at $50\,\mathrm{m\,s^{-1}}$ when the driver applies the brakes and gives the train a constant deceleration of magnitude $0.50\,\mathrm{m\,s^{-2}}$ for 100 s. Describe what happens to the train. Calculate the distance travelled by the train in 100 s. [7]

3 A boy stands on a cliff edge and throws a stone vertically upwards at time $t = 0$. The stone leaves his hand at $20\,\mathrm{m\,s^{-1}}$. Take the acceleration of the ball as $9.81\,\mathrm{m\,s^{-2}}$.

  a Show that the equation for the displacement of the ball is:

  $s = 20t - 4.9t^2$ [2]

  b What is the height of the stone 2.0 s after release and 6.0 s after release? [3]

  c When does the stone return to the level of the boy's hand? Assume the boy's hand does not move vertically after the ball is released. [4]

4 The graph in Figure 2.33 shows the variation of velocity with time of two cars, A and B, which are travelling in the same direction over a period of time of 40 s. Car A, travelling at a constant velocity of 40 m s^{-1}, overtakes car B at time t = 0. In order to catch up with car A, car B immediately accelerates uniformly for 20 s to reach a constant velocity of 50 m s^{-1}. Calculate:

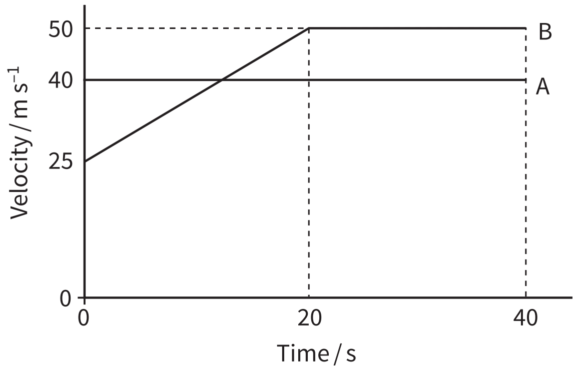


Figure 2.33 Velocity–time graphs for two cars, A and B. For End-of-chapter Question 4.

- **a** how far A travels during the first 20 s [2]
- **b** the acceleration and distance of travel of B during the first 20 s [5]
- **c** the additional time taken for B to catch up with A [2]
- **d** the distance each car will have then travelled since t = 0. [2]

5 An athlete competing in the long jump leaves the ground with a velocity of 5.6 m s^{-1} at an angle of 30° to the horizontal.

- **a** Determine the vertical component of the velocity and use this value to find the time between leaving the ground and landing. [2]
- **b** Determine the horizontal component of the velocity and use this value to find the horizontal distance travelled. [4]

6 Figure 2.34 shows an arrangement used to measure the acceleration of a metal plate as it falls vertically. The metal plate is released from rest and falls a distance of 0.200 m before breaking light beam 1. It then falls a further 0.250 m before breaking light beam 2.

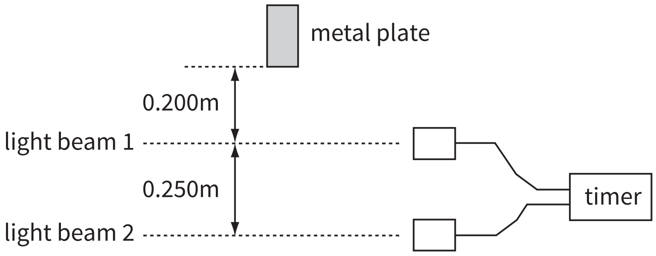


Figure 2.34 For End-of-chapter Question 6.

- **a** Calculate the time taken for the plate to fall 0.200 m from rest. (You may assume that the metal plate falls with an acceleration equal to the acceleration of free fall.) [2]
- **b** The timer measures the speed of the metal plate as it falls through each light beam. The speed as it falls through light beam 1 is 1.92 m s^{-1} and the speed as it falls through light beam 2 is 2.91 m s^{-1}.
  - **i** Calculate the acceleration of the plate between the two light beams. [2]
  - **ii** State and explain **one** reason why the acceleration of the plate is not equal to the acceleration of free fall. [2]

7 Figure 2.35 shows the velocity–time graph for a vertically bouncing ball. The ball is released at A and strikes the ground at B. The ball leaves the ground at D and reaches its maximum height at E. The effects of air resistance can be neglected.

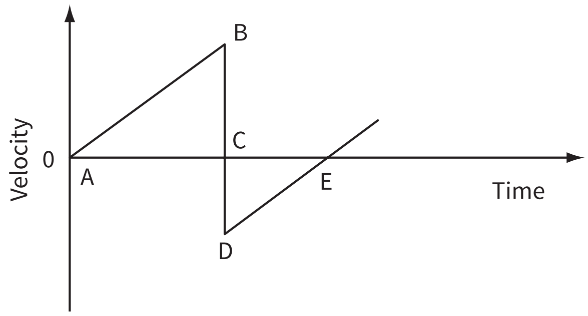


Figure 2.35 For End-of-chapter Question 7.

a State:

i why the velocity at D is negative [1]

ii why the gradient of the line AB is the same as the gradient of line DE [1]

iii what is represented by the area between the line AB and the time axis [1]

iv why the area of triangle ABC is greater than the area of triangle CDE. [1]

b The ball is dropped from rest from an initial height of 1.2 m. After hitting the ground the ball rebounds to a height of 0.80 m. The ball is in contact with the ground between B and D for a time of 0.16 s.

Using the acceleration of free fall, calculate:

i the speed of the ball immediately before hitting the ground [2]

ii the speed of the ball immediately after hitting the ground [2]

iii the acceleration of the ball while it is in contact with the ground. State the direction of this acceleration. [3]

8 A student measures the speed *v* of a trolley as it moves down a slope. The variation of *v* with time *t* is shown in the graph in Figure 2.36.

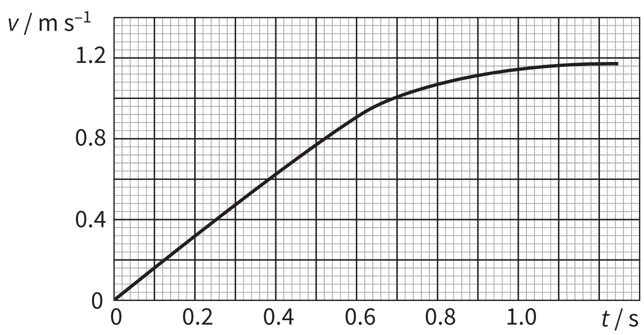


Figure 2.36 For End-of-chapter Question 8.

a Use the graph to find the acceleration of the trolley when $t = 0.7$ s. [2]

b State how the acceleration of the trolley varies between $t = 0$ and $t = 1.0$ s. Explain your answer by reference to the graph. [3]

c Determine the distance travelled by the trolley between $t = 0.6$ and $t = 0.8$ s. [3]

d The student obtained the readings for *v* using a motion sensor. The readings may have random errors and systematic errors. Explain how these two types of error affect the velocity–time graph. [2]

9 A car driver is travelling at speed v on a straight road. He comes over the top of a hill to find a fallen tree on the road ahead. He immediately brakes hard but travels a distance of 60 m at speed v before the brakes are applied. The skid marks left on the road by the wheels of the car are of length 140 m (Figure 2.37). The police investigate whether the driver was speeding and establish that the car decelerates at 2.0 m s^{-2} during the skid.

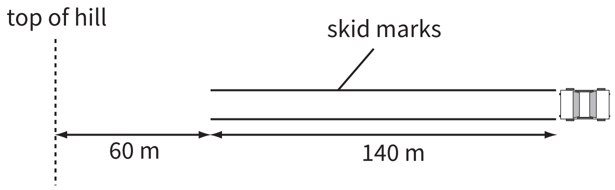


Figure 2.37 For End-of-chapter Question 9.

a Determine the initial speed v of the car before the brakes are applied. [2]

b Determine the time taken between the driver coming over the top of the hill and applying the brakes. Suggest whether this shows whether the driver was alert to the danger. [2]

c The speed limit on the road is 100 km/h. Determine whether the driver was breaking the speed limit. [2]

10 A hot-air balloon rises vertically. At time $t = 0$, a ball is released from the balloon. Figure 2.38 shows the variation of the ball's velocity v with t. The ball hits the ground at $t = 4.1$ s.

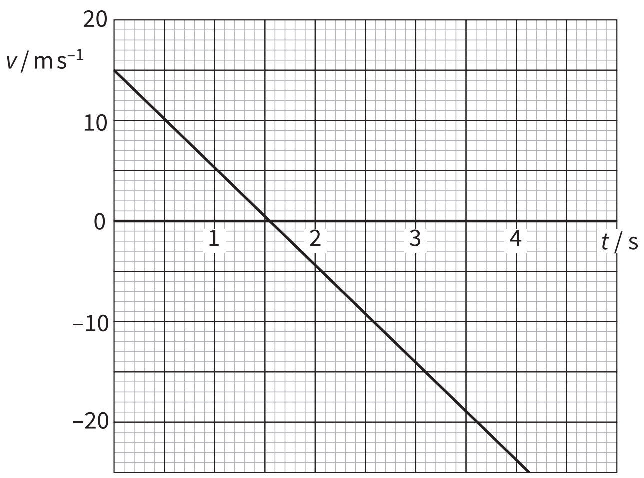


Figure 2.38 For End-of-chapter Question 10.

a Explain how the graph shows that the acceleration of the ball is constant. [1]

b Use the graph to:

i determine the time at which the ball reaches its highest point [1]

ii show that the ball rises for a further 12 m between release and its highest point [2]

iii determine the distance between the highest point reached by the ball and the ground. [2]

c The equation relating v and t is $v = 15 - 9.81t$. Explain the significance in the equation of:

i the number 15 [1]

ii the negative sign. [1]

11 An aeroplane is travelling horizontally at a speed of $80\,m\,s^{-1}$ and drops a crate of emergency supplies (Figure 2.39). To avoid damage, the maximum vertical speed of the crate on landing is $20\,m\,s^{-1}$. You may assume air resistance is negligible.

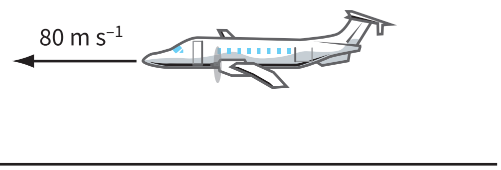
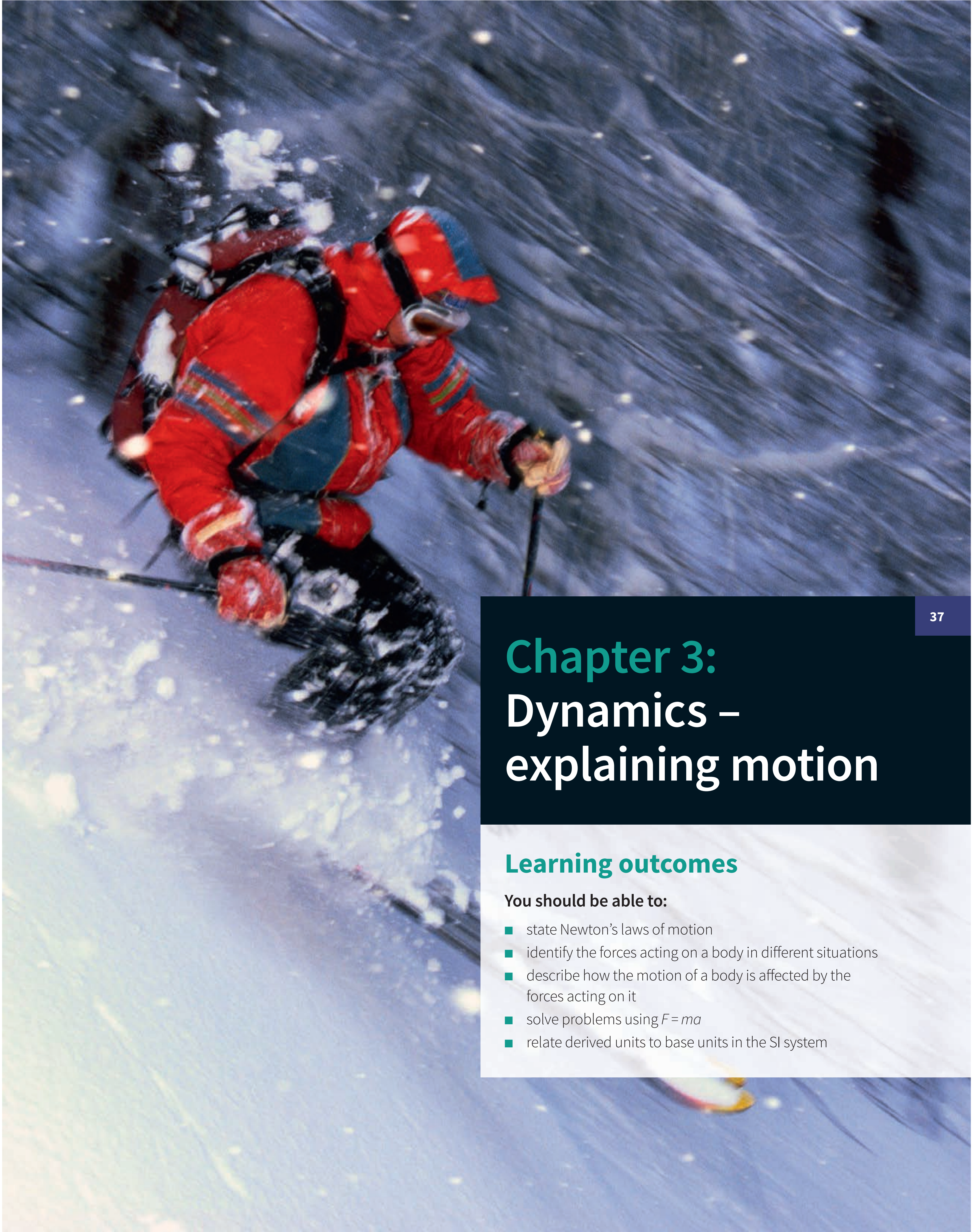


Figure 2.39 For End-of-chapter Question 11.

a Calculate the maximum height of the aeroplane when the crate is dropped. [2]

b Calculate the time taken for the crate to reach the ground from this height. [2]

c The aeroplane is travelling at the maximum permitted height. Calculate the horizontal distance travelled by the crate after it is released from the aeroplane. [1]



Chapter 3: Dynamics – explaining motion

Learning outcomes

You should be able to:

- state Newton's laws of motion
- identify the forces acting on a body in different situations
- describe how the motion of a body is affected by the forces acting on it
- solve problems using $F = ma$
- relate derived units to base units in the SI system

Force and acceleration

If you have ever flown in an aeroplane you will know how the back of the seat pushes you forwards when the aeroplane accelerates down the runway (Figure 3.1). The pilot must control many forces on the aeroplane to ensure a successful take-off.

In Chapters 1 and 2 we saw how motion can be described in terms of displacement, velocity, acceleration and so on. This is known as kinematics. Now we are going to look at how we can explain how an object moves in terms of the forces which change its motion. This is known as dynamics.

Figure 3.1 An aircraft takes off – the force provided by the engines causes the aircraft to accelerate.

Calculating the acceleration

Figure 3.2a shows how we represent the force which the motors on a train provide to cause it to accelerate. The resultant force is represented by a green arrow. The direction of the arrow shows the direction of the resultant force. The magnitude (size) of the resultant force of 20 000 N is also shown.

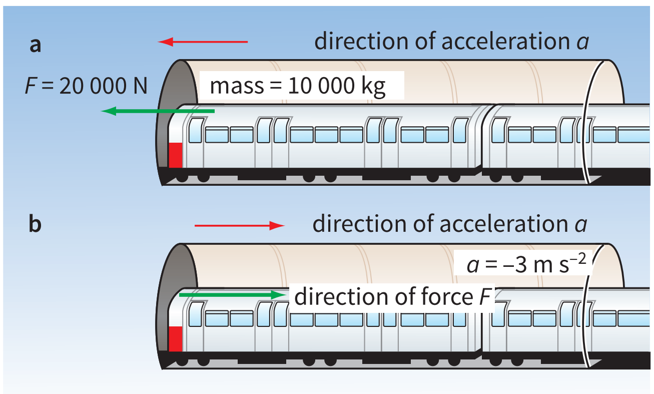


Figure 3.2 A force is needed to make the train **a** accelerate, and **b** decelerate.

To calculate the acceleration a of the train produced by the resultant force F, we must also know the train's mass m (Table 3.1). These quantities are related by:

$$a = \frac{F}{m} \quad \text{or} \quad F = ma$$

Quantity	Symbol	Unit
resultant force	F	N (newtons)
mass	m	kg (kilograms)
acceleration	a	$\mathrm{m\,s^{-2}}$ (metres per second squared)

Table 3.1 The quantities related by $F = ma$.

In this example we have $F = 20\,000$ N and $m = 10\,000$ kg, and so:

$$a = \frac{F}{m} = \frac{10\,000}{10\,000} = 2\ \mathrm{m\,s^{-2}}$$

In Figure 3.2b, the train is decelerating as it comes into a station. Its acceleration is $-3.0\ \mathrm{m\,s^{-2}}$. What force must be provided by the braking system of the train?

$$F = ma = 10\,000 \times -3 = -30\,000\ \mathrm{N}$$

The minus sign shows that the force must act towards the right in the diagram, in the opposite direction to the motion of the train.

Force, mass and acceleration

The equation we used above, $F = ma$, is a simplified version of **Newton's second law** of motion.

> For a body of constant mass, its acceleration is directly proportional to the resultant force applied to it.

An alternative form of Newton's second law is given in Chapter 6 when you have studied momentum. Since Newton's second law holds for objects that have a constant mass, this equation can be applied to a train whose mass remains constant during its journey. The equation $a = \frac{F}{m}$ relates acceleration, resultant force and mass. In particular, it shows that the bigger the force, the greater the acceleration it produces. You will probably feel that this is an unsurprising result. For a given object, the acceleration is directly proportional to the resultant force:

$$a \propto F$$

The equation also shows that the acceleration produced by a force depends on the mass of the object. The **mass** of an object is a measure of its **inertia**, or its ability to resist any change in its motion. The greater the mass, the smaller the acceleration which results. If you push your hardest against a small car (which has a small mass), you will have a greater effect than if you push against a more massive car (Figure 3.3). So, for a constant force, the acceleration is inversely proportional to the mass:

$$a \propto \frac{1}{m}$$

The train driver knows that, when the train is full during the rush hour, it has a smaller acceleration. This is because its mass is greater when it is full of people. Similarly, it is more difficult to stop the train once it is moving. The brakes must be applied earlier to avoid the train overshooting the platform at the station.

WORKED EXAMPLES

1 A cyclist of mass 60 kg rides a bicycle of mass 20 kg. When starting off, the cyclist provides a force of 200 N. Calculate the initial acceleration.

Step 1 This is a straightforward example. First, we must calculate the combined mass m of the bicycle and its rider:

$m = 20 + 60 = 80\,\text{kg}$

We are given the force F:

force causing acceleration $F = 200\,\text{N}$

Step 2 Substituting these values gives:

$$a = \frac{F}{m} = \frac{200}{80} = 2.5\,\text{m s}^{-2}$$

So the cyclist's acceleration is $2.5\,\text{m s}^{-2}$.

2 A car of mass 500 kg is travelling at $20\,\text{m s}^{-1}$. The driver sees a red traffic light ahead, and slows to a halt in 10 s. Calculate the braking force provided by the car.

Step 1 In this example, we must first calculate the acceleration required. The car's final velocity is $0\,\text{m s}^{-1}$, so its change in velocity $\Delta v = 0 - 20 = -20\,\text{m s}^{-1}$

$$\text{acceleration } a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$= \frac{\Delta v}{\Delta t} = \frac{-20}{10} = -2\,\text{ms}^{-2}$$

Step 2 To calculate the force, we use:

$F = ma = 500 \times -2 = -1000\,\text{N}$

So the brakes must provide a force of 1000 N. (The minus sign shows a force decreasing the velocity of the car.)

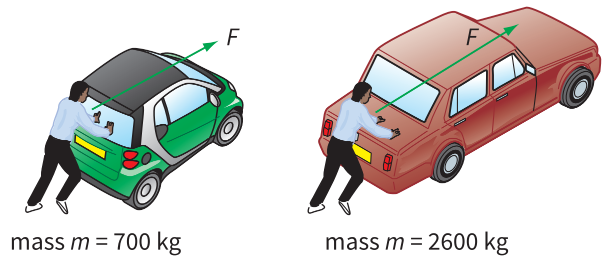


Figure 3.3 It is easier to make a small mass accelerate than a large mass.

QUESTIONS

1 Calculate the force needed to give a car of mass 800 kg an acceleration of $2.0\,\text{m s}^{-2}$.

2 A rocket has a mass of 5000 kg. At a particular instant, the resultant force acting on the rocket is 200 000 N. Calculate its acceleration.

3 (In this question, you will need to make use of the equations of motion which you studied in Chapter 2.) A motorcyclist of mass 60 kg rides a bike of mass 40 kg. As she sets off from the lights, the forward force on the bike is 200 N. Assuming the resultant force on the bike remains constant, calculate the bike's velocity after 5.0 s.

Understanding SI units

Any quantity that we measure or calculate consists of a value and a unit. In physics, we mostly use units from the SI system. These units are all defined with extreme care, and for a good reason. In science and engineering, every measurement must be made on the same basis, so that measurements obtained in different laboratories can be compared. This is important for commercial reasons, too. Suppose an engineering firm in Taiwan is asked to produce a small part for the engine of a car which is to be assembled in India. The dimensions are given in millimetres and the part must be made with an accuracy of a tiny fraction of a millimetre. All concerned must know that the part will fit correctly – it wouldn't be acceptable to use a different millimetre scale in Taiwan and India.

Engineering measurements, as well as many other technical measurements, are made using SI units to ensure that customers get what they expected (and can complain if they don't). So governments around the world have set up standards laboratories to ensure that measuring instruments are as accurate as is required – scales weigh correctly, police speed cameras give reliable measurements, and so on. (Other, non-SI, units such as the foot, pound or hour, are defined in terms of SI units.)

Base units, derived units

The metre, kilogram and second are three of the seven SI **base units**. These are defined with great precision so that every standards laboratory can reproduce them correctly.

Other units, such as units of speed ($m\,s^{-1}$) and acceleration ($m\,s^{-2}$) are known as **derived units** because they are combinations of base units. Some derived units, such as the newton and the joule, have special names which are more convenient to use than giving them in terms of base units. The definition of the newton will show you how this works.

Defining the newton

Isaac Newton (1642–1727) played a significant part in developing the scientific idea of force. Building on Galileo's earlier thinking, he explained the relationship between force, mass and acceleration, which we now write as $F = ma$. For this reason, the SI unit of force is named after him.

We can use the equation $F = ma$ to define the **newton** (N).

One newton is the force that will give a 1 kg mass an acceleration of $1\,m\,s^{-2}$ in the direction of the force.

$1\,N = 1\,kg \times 1\,m\,s^{-2}$ or $1\,N = 1\,kg\,m\,s^{-2}$

The seven base units

In mechanics (the study of forces and motion), the units we use are based on three base units: the metre, kilogram and second. As we move into studying electricity, we will need to add another base unit, the ampere. Heat requires another base unit, the kelvin (the unit of temperature).

Table 3.2 shows the seven base units of the SI system. Remember that all other units can be derived from these seven. The equations that relate them are the equations that you will learn as you go along (just as $F = ma$ relates the newton to the kilogram, metre and second). The unit of luminous intensity is not part of the A/AS course.

Base unit	Symbol	Base unit
length	*x*, *l*, *s* etc.	m (metre)
mass	*m*	kg (kilogram)
time	*t*	s (second)
electric current	*I*	A (ampere)
thermodynamic temperature	*T*	K (kelvin)
amount of substance	*n*	mol (mole)
luminous intensity	*I*	cd (candela)

Table 3.2 SI base quantities and units. In this course, you will learn about all of these except the candela.

QUESTION

4 The pull of the Earth's gravity on an apple (its weight) is about 1 newton. We could devise a new international system of units by defining our unit of force as the weight of an apple. State as many reasons as you can why this would not be a very useful definition.

Other SI units

Using only seven base units means that only this number of quantities have to be defined with great precision. There would be confusion and possible contradiction if more units were also defined. For example, if the density of water were **defined** as exactly $1\,g\,cm^{-3}$, then $1000\,cm^3$ of a sample of water would have a mass of exactly 1 kg. However, it is unlikely that the mass of this volume of water would equal exactly the mass of the standard kilogram. The standard kilogram, which is kept in France, is the one standard from which all masses can ultimately be measured.

All other units can be derived from the base units. This is done using the definition of the quantity. For example, speed is defined as $\frac{\text{distance}}{\text{time}}$, and so the base units of speed in the SI system are $\mathbf{m\,s^{-1}}$.

Since the defining equation for force is $F = ma$, the base units for force are $\mathbf{kg\,m\,s^{-2}}$.

Equations that relate different quantities must have the same base units on each side of the equation. If this does not happen the equation must be wrong.

When each term in an equation has the same base units the equation is said to be **homogeneous**.

QUESTIONS

5 Determine the base units of:

a pressure $(= \frac{\text{force}}{\text{area}})$

b energy (= force × distance)

c density $(= \frac{\text{mass}}{\text{volume}})$

6 Use base units to prove that the following equations are homogeneous.

a pressure
= density × acceleration due to gravity × depth

b distance travelled
= initial speed × time + $\frac{1}{2}$ acceleration × time^2
$(s = ut + \frac{1}{2}at^2)$

WORKED EXAMPLE

3 It is suggested that the time T for one oscillation of a swinging pendulum is given by the equation $T^2 = 4\pi^2(l/g)$ where l is the length of the pendulum and g is the acceleration due to gravity. Show that this equation is homogeneous.

For the equation to be homogeneous, the term on the left-hand side must have the same base units as all the terms on the right-hand side.

Step 1 The base unit of time T is **s**. The base unit of the left-hand side of the equation is therefore $\mathbf{s^2}$.

Step 2 The base unit of l is **m**. The base units of g are $\mathrm{m\,s^{-2}}$. Therefore the base unit of the right-hand side is $\frac{\mathbf{m}}{(\mathbf{m\,s^{-2}})} = \mathbf{s^2}$. (Notice that the constant $4\pi^2$ has no units.)

Since the base units on the left-hand side of the equation are the same as those on the right, the equation is homogeneous.

Prefixes

Each unit in the SI system can have **multiples** and **sub-multiples** to avoid using very high or low numbers. For example 1 millimetre (mm) is one thousandth of a metre and 1 micrometre (μm) is one millionth of a metre.

The **prefix** comes before the unit. In the unit mm, the first m is the prefix milli and the second m is the unit metre. You will need to recognise a number of prefixes for the A/AS course, as shown in Table 3.3.

Multiples			Sub-multiples		
Multiple	**Prefix**	**Symbol**	**Multiple**	**Prefix**	**Symbol**
10^3	kilo	k	10^{-1}	deci	d
10^6	mega	M	10^{-2}	centi	c
10^9	giga	G	10^{-3}	mill	m
10^{12}	tera	T	10^{-6}	micro	μ
10^{15}	peta	P	10^{-9}	nano	n
			10^{-12}	pico	p

Table 3.3 Multiples and sub-multiples.

You must take care when using prefixes.

- Squaring or cubing prefixes – for example:
  $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$
  so $1\,\mathrm{cm}^2 = (10^{-2}\,\mathrm{m})^2 = 10^{-4}\,\mathrm{m}^2$
  and $1\,\mathrm{cm}^3 = (10^{-2}\,\mathrm{m})^3 = 10^{-6}\,\mathrm{m}^3$.
- Writing units – for example, you must leave a small space between each unit when writing a speed such as $3\,\mathrm{m\,s^{-1}}$, because if you write it as $3\,\mathrm{ms^{-1}}$ it would mean 3 millisecond^{-1}.

QUESTIONS

7 Find the area of one page of this book in cm^2 and then convert your value to m^2.

8 Write down in powers of ten the values of the following quantities:

a 60 pA

b 500 MW

c 20 000 mm

WORKED EXAMPLE

4 The density of water is $1.0\,\mathrm{g\,cm^{-3}}$. Calculate this value in $\mathrm{kg\,m^{-3}}$.

Step 1 Find the conversions for the units:

$1\,\mathrm{g} = 1 \times 10^{-3}\,\mathrm{kg}$

$1\,\mathrm{cm}^3 = 1 \times 10^{-6}\,\mathrm{m}^3$

Step 2 Use these in the value for the density of water:

$$1.0\,\mathrm{g\,cm^{-3}} = \frac{1.0 \times 1 \times 10^{-3}}{1 \times 10^{-6}}$$

$$= 1.0 \times 10^3\,\mathrm{kg\,m^{-3}}$$

The pull of gravity

Now we need to consider some specific forces – such as weight and friction.

When Isaac Newton was confined to his rural home to avoid the plague which was rampant in other parts of England, he is said to have noticed an apple fall to the ground. From this, he developed his theory of gravity which relates the motion of falling objects here on Earth to the motion of the Moon around the Earth, and the planets around the Sun.

The force which caused the apple to accelerate was the pull of the Earth's gravity. Another name for this force is the **weight** of the apple. The force is shown as an arrow, pulling vertically downwards on the apple (Figure 3.4). It is usual to show the arrow coming from the centre of the apple – its **centre of gravity**. The centre of gravity of an object is defined as the point where its entire weight appears to act.

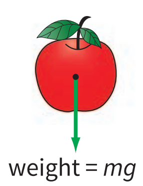


Figure 3.4 The weight of an object is a force caused by the Earth's gravity. It acts vertically down on the object.

Large and small

A large rock has a greater weight than a small rock, but if you push both rocks over a cliff at the same time, they will fall at the same rate. In other words, they have the **same** acceleration, regardless of their mass. This is a surprising result. Common sense may suggest that a heavier object will fall faster than a lighter one. It is said that Galileo dropped a large cannon ball and a small cannon ball from the top of the Leaning Tower of Pisa in Italy, and showed that they landed simultaneously. He may never actually have done this, but the story illustrates that the result is not intuitively obvious – if everyone thought that the two cannon balls would accelerate at the same rate, there would not have been any experiment or story.

In fact, we are used to lighter objects falling more slowly than heavy ones. A feather drifts down to the floor, while a stone falls quickly. However, we are being misled by the presence of **air resistance**. The force of air resistance has a large effect on the falling feather, and almost no effect on the falling stone. When astronauts visited the Moon (where there is virtually no atmosphere and so no air resistance), they were able to show that a feather and a stone fell side-by-side to the ground.

As we saw in Chapter 2, an object falling freely close to the Earth's surface has an acceleration of roughly $9.81\ \text{m s}^{-2}$, the acceleration of free fall g.

We can find the force causing this acceleration using $F = ma$. This force is the object's **weight**. Hence the weight W of an object is given by:

weight = mass × acceleration of free fall

or

$$W = mg$$

Gravitational field strength

Here is another way to think about the significance of g. This quantity indicates how strong gravity is at a particular place. The Earth's gravitational field is stronger than the Moon's. On the Earth's surface, gravity gives an acceleration of free fall of about $9.81\ \text{m s}^{-2}$. On the Moon, gravity is weaker; it only gives an acceleration of free fall of about $1.6\ \text{m s}^{-2}$. So g indicates the strength of the gravitational field at a particular place:

g = gravitational field strength

and

weight = mass × gravitational field strength

(Gravitational field strength has units of N kg^{-1}. This unit is equivalent to m s^{-2}.)

QUESTION

9 Estimate the mass and weight of each of the following at the surface of the Earth:

- **a** a kilogram of potatoes
- **b** this book
- **c** an average student
- **d** a mouse
- **e** a 40-tonne truck.

(For estimates, use $g = 10\ \text{m s}^{-2}$; 1 tonne = 1000 kg.)

On the Moon

The Moon is smaller and has less mass than the Earth, and so its gravity is weaker. If you were to drop a stone on the Moon, it would have a smaller acceleration. Your hand is about 1 m above ground level; a stone takes about 0.45 s to fall through this distance on the Earth, but about 1.1 s on the surface of the Moon. The acceleration of free fall on the Moon is about one-sixth of that on the Earth:

$$g_{\text{Moon}} = 1.6\ \text{m s}^{-2}$$

It follows that objects weigh less on the Moon than on the Earth. They are not completely weightless, because the Moon's gravity is not zero.

Mass and weight

We have now considered two related quantities, mass and weight. It is important to distinguish carefully between these (Table 3.4).

If your moon-buggy breaks down (Figure 3.5), it will be no easier to get it moving on the Moon than on the Earth. This is because its mass does not change, because it is made from just the same atoms and molecules wherever it is. From $F = ma$, it follows that if m doesn't change, you will need the same force F to start it moving.

However, your moon-buggy will be easier to lift on the Moon, because its weight will be less. From $W = mg$, since g is less on the Moon, it has a smaller weight than when on the Earth.

Quantity	Symbol	Unit	Comment
mass	m	kg	this does not vary from place to place
weight	mg	N	this a force – it depends on the strength of gravity

Table 3.4 Distinguishing between mass and weight.

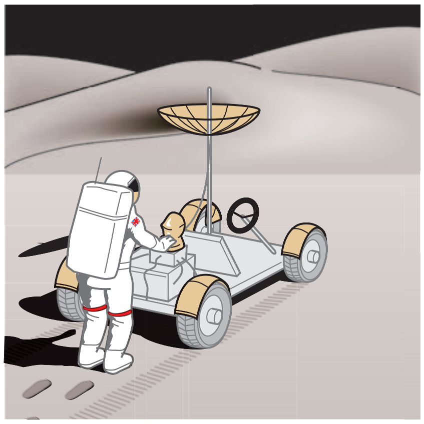


Figure 3.5 The mass of a moon-buggy is the same on the Moon as on the Earth, but its weight is smaller.

Mass and inertia

It took a long time for scientists to develop correct ideas about forces and motion. We will start by thinking about some wrong ideas, and then consider why Galileo, Newton and others decided new ideas were needed.

Observations and ideas

Here are some observations to think about:

- The large tree trunk shown in Figure 3.6 is being dragged from a forest. The elephant provides the force needed to pull it along. If the elephant stops pulling, the tree trunk will stop moving.
- A horse is pulling a cart. If the horse stops pulling, the cart soon stops.
- You are riding a bicycle. If you stop pedalling, the bicycle will come to a halt.
- You are driving along the road. You must keep your foot on the accelerator pedal, otherwise the car will not keep moving.
- You kick a football. The ball rolls along the ground and gradually stops.

In each of these cases, there is a force which makes something move – the pull of the elephant or the horse, your push on the bicycle pedals, the force of the car engine, the push of your foot. Without the force, the moving object comes to a halt. So what conclusion might we draw?

Figure 3.6 An elephant provides the force needed to drag this tree from the forest.

A moving object needs a force to keep it moving.

This might seem a sensible conclusion to draw, but it is wrong. We have not thought about all the forces involved. The missing force is friction.

In each example above, friction (or air resistance) makes the object slow down and stop when there is no force pushing or pulling it forwards. For example, if you stop pedalling your cycle, air resistance will slow you down. There is also friction at the axles of the wheels, and this too will slow you down. If you could lubricate your axles and cycle in a vacuum, you could travel along at a steady speed for ever, without pedalling!

In the 17th century, astronomers began to use telescopes to observe the night sky. They saw that objects such as the planets could move freely through space. They simply kept on moving, without anything providing a force to push them. Galileo came to the conclusion that this was the natural motion of objects.

- An object at rest will stay at rest, unless a force causes it to start moving.
- A moving object will continue to move at a steady speed in a straight line, unless a force acts on it.

So objects move with a constant velocity, unless a force acts on them. (Being stationary is simply a particular case of this, where the velocity is zero.) Nowadays it is much easier to appreciate this law of motion, because we have more experience of objects moving with little or no friction – roller-skates with low-friction bearings, ice skates, and spacecraft in empty space. In Galileo's day, people's everyday experience was of dragging things along the ground, or pulling things on carts with high-friction axles. Before Galileo, the orthodox scientific idea was that a force must act all the time to keep an object moving – this had been handed down from the time of the ancient Greek philosopher Aristotle. So it was a great achievement when scientists were able to develop a picture of a world without friction.

The idea of inertia

The tendency of a moving object to carry on moving is sometimes known as **inertia**.

- An object with a large mass is difficult to stop moving – think about catching a cricket ball, compared with a tennis ball.
- Similarly, a stationary object with a large mass is difficult to start moving – think about pushing a car to get it started.
- It is difficult to make a massive object change direction – think about the way a fully laden supermarket trolley tries to keep moving in a straight line.

All of these examples suggest another way to think of an object's mass; it is a measure of its inertia – how difficult it is to change the object's motion. Uniform motion is the natural state of motion of an object. Here, **uniform motion** means 'moving with constant velocity' or 'moving at a steady speed in a straight line'. Now we can summarise these findings as **Newton's first law of motion**.

An object will remain at rest or in a state of uniform motion unless it is acted on by a resultant force.

In fact, this is already contained in the simple equation we have been using to calculate acceleration, $F = ma$. If no resultant force acts on an object ($F = 0$), it will not accelerate ($a = 0$). The object will either remain stationary or it will continue to travel at a constant velocity. If we rewrite the equation as $a = \frac{F}{m}$, we can see that the greater the mass m, the smaller the acceleration a produced by a force F.

QUESTIONS

10 Use the idea of inertia to explain why some large cars have power-assisted brakes.

11 A car crashes head-on into a brick wall. Use the idea of inertia to explain why the driver is more likely to come out through the windscreen if he or she is not wearing a seat belt.

Top speed

The vehicle shown in Figure 3.7 is capable of speeds as high as 760 mph, greater than the speed of sound. Its streamlined shape is designed to cut down air resistance and its jet engines provide a strong forward force to accelerate it up to top speed. All vehicles have a top speed.

Figure 3.7 The Thrust SSC rocket car broke the world land-speed record in 1997. It achieved a top speed of 763 mph (just over 340 m s^{-1}) over a distance of 1 mile (1.6 km).

But why can't they go any faster? Why can't a car driver keep pressing on the accelerator pedal, and simply go faster and faster?

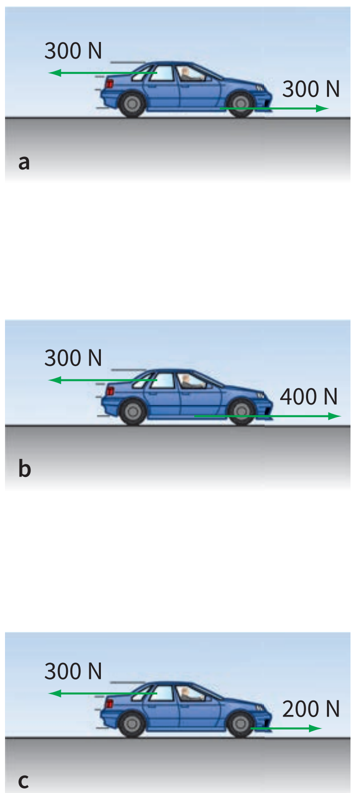
To answer this, we have to think about the two forces mentioned above: air resistance and the forward thrust (force) of the engine. The vehicle will accelerate so long as the thrust is greater than the air resistance. When the two forces are equal, the resultant force on the vehicle is zero, and the vehicle moves at a steady velocity.

Balanced and unbalanced forces

If an object has two or more forces acting on it, we have to consider whether or not they are 'balanced' (Figure 3.8). Forces on an object are balanced when the resultant force on the object is zero. The object will either remain at rest or have a constant velocity.

We can calculate the **resultant force** by adding up two (or more) forces which act in the same straight line. We must take account of the direction of each force. In the examples in Figure 3.8, forces to the right are positive and forces to the left are negative.

When a car travels slowly, it encounters little air resistance. However, the faster it goes, the more air it has to push out of the way each second, and so the greater



a: Two equal forces acting in opposite directions cancel each other out. We say they are **balance** The car will continue to move at a steady velocit in a straight line.
resultant force = 0 N

b: These two forces are unequal, so they do not cancel out. They are **unbalanced**. The car will accelerate.
resultant force
= 400 N – 300
= 100 N to the **right**

c: Again the forces are unbalanced. This time, the car will slow down o decelerate.
resultant force
= 400 N – 300 N
= 100 N to the **left**

Figure 3.8 Balanced and unbalanced forces.

the air resistance. Eventually the backward force of air resistance equals the forward force provided between the tyres and the road, and the forces on the car are balanced. It can go no faster – it has reached its top speed.

Free fall

Skydivers (Figure 3.9) are rather like cars – at first, they accelerate freely. At the start of the fall, the only force acting on the diver is his or her weight. The acceleration of the diver at the start must therefore be *g*. Then increasing air resistance opposes their fall and their acceleration decreases. Eventually they reach a maximum velocity, known as the **terminal velocity**. At the terminal velocity the air resistance is equal to the weight. The terminal velocity is approximately 120 miles per hour (about 50 m s^{-1}), but it depends on the diver's weight and orientation. Head-first is fastest.

Figure 3.9 A skydiver falling freely.

The idea of a parachute is to greatly increase the air resistance. Then terminal velocity is reduced, and the parachutist can land safely. Figure 3.10 shows how a parachutist's velocity might change during descent.

Terminal velocity depends on the weight and surface area of the object. For insects, air resistance is much greater relative to their weight than for a human being and so their terminal velocity is quite low. Insects can be swept up several kilometres into the atmosphere by rising air streams. Later, they fall back to Earth uninjured. It is said that mice can survive a fall from a high building for the same reason.

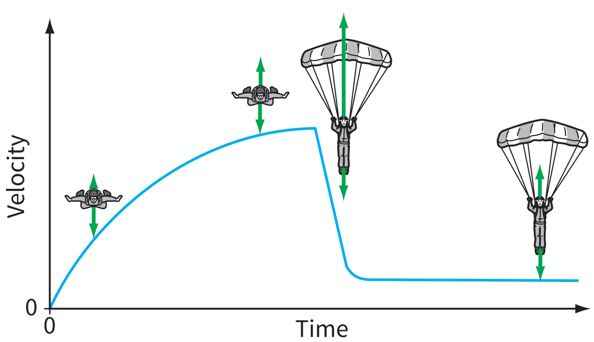


Figure 3.10 The velocity of a parachutist varies during a descent. The force arrows show weight (downwards) and air resistance (upwards).

Moving through fluids

Air resistance is just one example of the resistive or **viscous forces** which objects experience when they move through a fluid – a liquid or a gas. If you have ever run down the beach and into the sea, or tried to wade quickly through the water of a swimming pool, you will have experienced the force of **drag**. The deeper the water gets, the more it resists your movement and the harder you have to work to make progress through it. In deep water, it is easier to swim than to wade.

You can observe the effect of drag on a falling object if you drop a key or a coin into the deep end of a swimming pool. For the first few centimetres, it speeds up, but for the remainder of its fall, it has a steady speed. (If it fell through the same distance in air, it would accelerate all the way.) The drag of water means that the falling object reaches its terminal velocity very soon after it is released. Compare this with a skydiver, who has to fall hundreds of metres before reaching terminal velocity.

Moving through air

We rarely experience drag in air. This is because air is much less dense than water; its density is roughly $\frac{1}{800}$ that of water. At typical walking speed, we do not notice the effects of drag. However, if you want to move faster, they can be important. Racing cyclists, like the one shown in Figure 3.11, wear tight-fitting clothing and streamlined

Figure 3.11 A racing cyclist adopts a posture which helps to reduce drag. Clothing, helmet and even the cycle itself are designed to allow them to go as fast as possible.

helmets. Other athletes may take advantage of the drag of air. The runner in Figure 3.12 is undergoing resistance training. The parachute provides a backward force against which his muscles must work. This should help to develop his muscles.

Figure 3.12 A runner making use of air resistance to build up his muscles.

QUESTIONS

12 If you drop a large stone and a small stone from the top of a tall building, which one will reach the ground first? Explain your answer.

13 In a race, downhill skiers want to travel as quickly as possible. They are always looking for ways to increase their top speed. Explain how they might do this. Think about:

- **a** their skis
- **b** their clothing
- **c** their muscles
- **d** the slope.

14 Skydivers jump from a plane at intervals of a few seconds. If two divers wish to join up as they fall, the second must catch up with the first.

- **a** If one diver is more massive than the other, which should jump first? Use the idea of forces and terminal velocity to explain your answer.
- **b** If both divers are equally massive, suggest what the second might do to catch up with the first.

WORKED EXAMPLES

5 A car of mass 500 kg is travelling along a flat road. The forward force provided between the car tyres and the road is 300 N and the air resistance is 200 N. Calculate the acceleration of the car.

Step 1 Start by drawing a diagram of the car, showing the forces mentioned in the question (Figure 3.13). Calculate the resultant force on the car; the force to the right is taken as positive:

resultant force = 300 − 200 = 100 N

Step 2 Now use $F = ma$ to calculate the car's acceleration:

$$a = \frac{F}{m} = \frac{100}{500} = 0.20\ \mathrm{m\,s^{-2}}$$

So the car's acceleration is $0.20\ \mathrm{m\,s^{-2}}$.

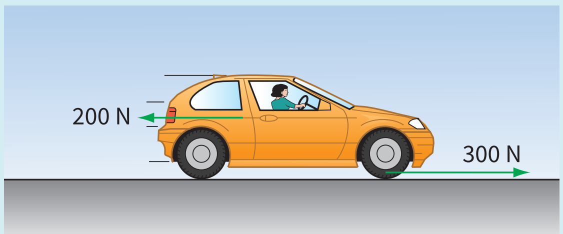


Figure 3.13 The forces on an accelerating car.

6 The maximum forward force a car can provide is 500 N. The air resistance F which the car experiences depends on its speed according to $F = 0.2v^2$, where v is the speed in $\mathrm{m\,s^{-1}}$. Determine the top speed of the car.

Step 1 From the equation $F = 0.2v^2$, you can see that the air resistance increases as the car goes faster. Top speed is reached when the forward force equals the air resistance. So, at top speed:

$500 = 0.2v^2$

Step 2 Rearranging gives:

$$v^2 = \frac{500}{0.2} = 2500$$

$v = 50\ \mathrm{m\,s^{-1}}$

So the car's top speed is $50\ \mathrm{m\,s^{-1}}$ (this is about $180\ \mathrm{km\,h^{-1}}$).

Identifying forces

It is important to be able to identify the forces which act on an object. When we know what forces are acting, we can predict how it will move. Figure 3.14 shows some important forces, how they arise, and how we represent them in diagrams.

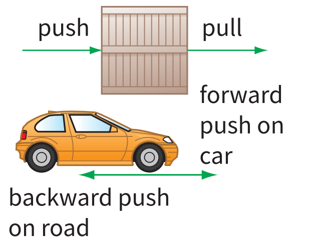
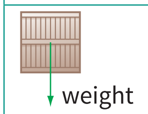
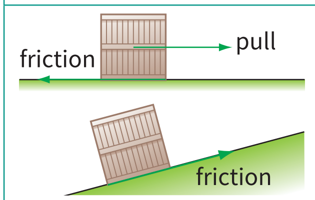
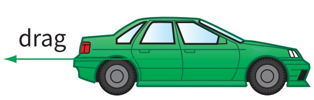
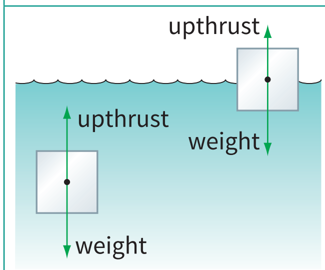
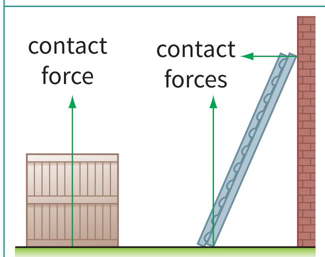
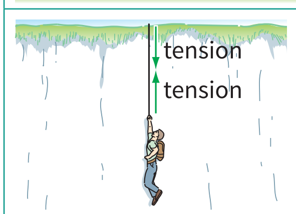
Diagram	Force	Important situations
	**Pushes and pulls.** You can make an object accelerate by pushing and pulling it. Your force is shown by an arrow pushing (or pulling) the object. The engine of a car provides a force to push backwards on the road. Frictional forces from the road on the tyre push the car forwards.	■ pushing and pulling ■ lifting ■ force of car engine ■ attraction and repulsion by magnets and by electric charges
	**Weight.** This is the force of gravity acting on the object. It is usually shown by an arrow pointing vertically downwards from the object's centre of gravity.	■ any object in a gravitational field ■ less on the Moon
	**Friction.** This is the force which arises when two surfaces rub over one another. If an object is sliding along the ground, friction acts in the opposite direction to its motion. If an object is stationary, but tending to slide – perhaps because it is on a slope – the force of friction acts up the slope to stop it from sliding down. Friction always acts along a surface, never at an angle to it.	■ pulling an object along the ground ■ vehicles cornering or skidding ■ sliding down a slope
	**Drag.** This force is similar to friction. When an object moves through air, there is friction between it and the air. Also, the object has to push aside the air as it moves along. Together, these effects make up drag. Similarly, when an object moves through a liquid, it experiences a drag force. Drag acts to oppose the motion of an object; it acts in the opposite direction to the object's velocity. It can be reduced by giving the object a streamlined shape.	■ vehicles moving ■ aircraft flying ■ parachuting ■ objects falling through air or water ■ ships sailing
	**Upthrust.** Any object placed in a fluid such as water or air experiences an upwards force. This is what makes it possible for something to float in water. Upthrust arises from the pressure which a fluid exerts on an object. The deeper you go, the greater the pressure. So there is more pressure on the lower surface of an object than on the upper surface, and this tends to push it upwards. If upthrust is greater than the object's weight, it will float up to the surface.	■ boats and icebergs floating ■ people swimming ■ divers surfacing ■ a hot air balloon rising
	**Contact force.** When you stand on the floor or sit on a chair, there is usually a force which pushes up against your weight, and which supports you so that you do not fall down. The contact force is sometimes known as the normal reaction of the floor or chair. (In this context, normal means 'perpendicular'.) The contact force always acts at right angles to the surface which produces it. The floor pushes straight upwards; if you lean against a wall, it pushes back against you horizontally.	■ standing on the ground ■ one object sitting on top of another ■ leaning against a wall ■ one object bouncing off another
	**Tension.** This is the force in a rope or string when it is stretched. If you pull on the ends of a string, it tends to stretch. The tension in the string pulls back against you. It tries to shorten the string. Tension can also act in springs. If you stretch a spring, the tension pulls back to try to shorten the spring. If you squash (compress) the spring, the tension acts to expand the spring.	■ pulling with a rope ■ squashing or stretching a spring

Figure 3.14 Some important forces.

Contact forces and upthrust

We will now think about the forces which act when two objects are in contact with each other. When two objects touch each other, each exerts a force on the other. These are called **contact forces**. For example, when you stand on the floor (Figure 3.15), your feet push downwards on the floor and the floor pushes back upwards on your feet. This is a vital force – the upward push of the floor prevents you from falling downwards under the pull of your weight.

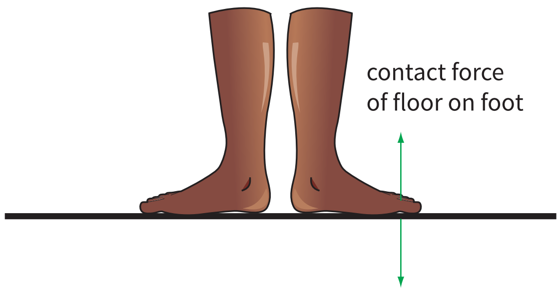


Figure 3.15 Equal and opposite contact forces act when you stand on the floor.

Where do these contact forces come from? When you stand on the floor, the floor becomes slightly compressed. Its atoms are pushed slightly closer together, and the interatomic forces push back against the compressing force. At the same time, the atoms in your feet are also pushed together so that they push back in the opposite direction. (It is hard to see the compression of the floor when you stand on it, but if you stand on a soft material such as foam rubber or a mattress you will be able to see the compression clearly.)

You can see from Figure 3.15 that the two contact forces act in opposite directions. They are also equal in magnitude. As we will see shortly, this is a consequence of Newton's third law of motion.

When an object is immersed in a fluid (a liquid or a gas), it experiences an upward force called **upthrust**. It is the upthrust of water which keeps a boat floating (Figure 3.16) and the upthrust of air which lifts a hot air balloon upwards.

The upthrust of water on a boat can be thought of as the contact force of the water on the boat. It is caused by the pressure of the water pushing upwards on the boat. Pressure arises from the motion of the water molecules colliding with the boat and the net effect of all these collisions is an upward force.

An object in air, such as a ball, has a very small upthrust acting on it, because the density of the air around it is low. Molecules hit the top surface of the ball pushing down, but only a few more molecules push upwards on the bottom of the ball, so the resultant force upwards, or the upthrust is low. If the ball is falling, air resistance is greater than this small upthrust but both these forces are acting upwards on the ball.

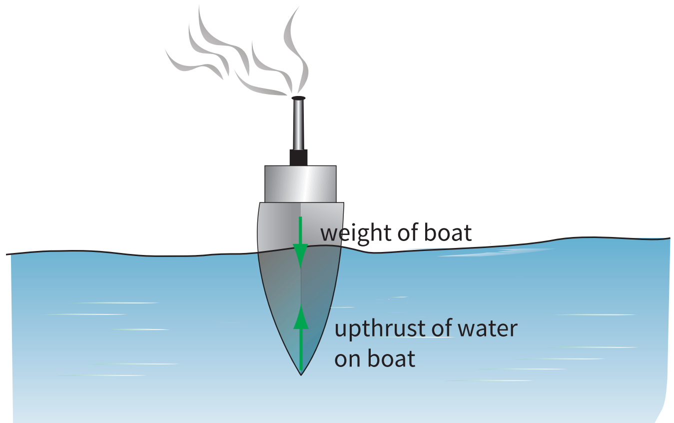


Figure 3.16 Without sufficient upthrust from the water, the boat would sink.

QUESTIONS

15 Name these forces:

- **a** the upward push of water on a submerged object
- **b** the force which wears away two surfaces as they move over one another
- **c** the force which pulled the apple off Isaac Newton's tree
- **d** the force which stops you falling through the floor
- **e** the force in a string which is holding up an apple
- **f** the force which makes it difficult to run through shallow water.

16 Draw a diagram to show the forces which act on a car as it travels along a level road at its top speed.

17 Imagine throwing a shuttlecock straight up in the air. Air resistance is more important for shuttlecocks than for a tennis ball. Air resistance always acts in the opposite direction to the velocity of an object.

Draw diagrams to show the two forces, weight and air resistance, acting on the shuttlecock:

- **a** as it moves upwards
- **b** as it falls back downwards.

Newton's third law of motion

For completeness, we should now consider **Newton's third law of motion**. (There is more about this in Chapter 6.)

When two objects interact, each exerts a force on the other. Newton's third law says that these forces are equal and opposite to each other:

> When two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction.

(These two forces are sometimes described as **action** and **reaction**, but this is misleading as it sounds as though one force arises as a consequence of the other. In fact, the two forces appear at the same time and we can't say that one caused the other.)

The two forces which make up a 'Newton's third law pair' have the following characteristics:

- They act on **different** objects.
- They are equal in magnitude.
- They are opposite in direction.
- They are forces **of the same type**.

What does it mean to say that the forces are 'of the same type'? We need to think about the type of interaction which causes the forces to appear.

- Two objects may attract each other because of the gravity of their masses – these are gravitational forces.
- Two objects may attract or repel because of their electrical charges – electrical forces.
- Two objects may touch – contact forces.
- Two objects may be attached by a string and pull on each other – tension forces.
- Two objects may attract or repel because of their magnetic fields – magnetic forces.

Figure 3.17 shows a person standing on the Earth's surface. The two gravitational forces are a Newton's third law pair, as are the two contact forces. Don't be misled into thinking that the person's weight and the contact force of the floor are a Newton's third law pair. Although they are 'equal and opposite', they do not act on different objects and they are not of the same type.

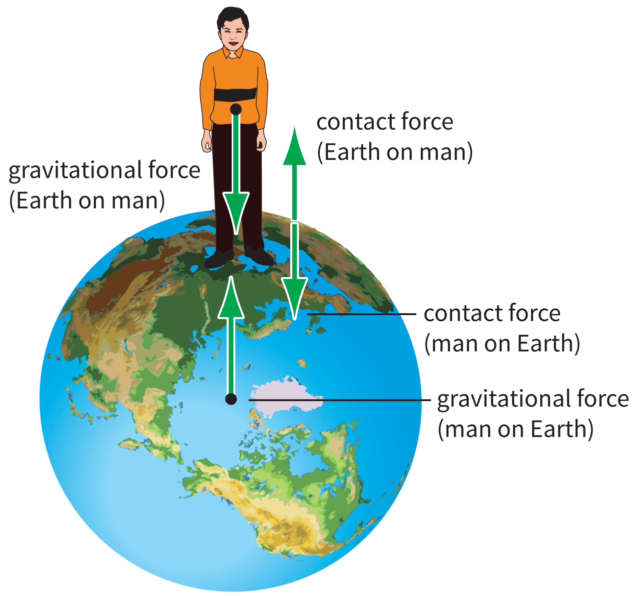


Figure 3.17 For each of the forces that the Earth exerts on you, an equal and opposite force acts on the Earth.

QUESTION

18 Describe one 'Newton's third law pair' of forces involved in the following situations. In each case, state the object that each force acts on, the type of force and the direction of the force.

a You step on someone's toe.

b A car hits a brick wall and comes to rest.

c A car slows down by applying the brakes.

d You throw a ball upwards into the air.

Summary

- An object will remain at rest or in a state of uniform motion unless it is acted on by an external force. This is Newton's first law of motion.
- For a body of constant mass, the acceleration is directly proportional to the resultant force applied to it. Resultant force F, mass m and acceleration a are related by the equation:

  resultant force = mass × acceleration

  $F = ma$

  This is a form of Newton's second law of motion.
- When two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction.
  This is Newton's third law of motion.
- The acceleration produced by a force is in the same direction as the force. Where there are two or more forces, we must determine the resultant force.
- A newton (N) is the force required to give a mass of 1 kg an acceleration of $1\ \mathrm{m\,s^{-2}}$ in the direction of the force.
- The greater the mass of an object, the more it resists changes in its motion. Mass is a measure of the object's inertia.
- The weight of an object is a result of the pull of gravity on it:

  weight = mass × acceleration of free fall ($W = mg$)

  weight = mass × gravitational field strength
- An object falling freely under gravity has a constant acceleration provided the gravitational field strength is constant. However, fluid resistance (such as air resistance) reduces its acceleration. Terminal velocity is reached when the fluid resistance is equal to the weight of the object.

End-of-chapter questions

1 When a golfer hits a ball his club is in contact with the ball for about 0.0005 s and the ball leaves the club with a speed of 70 m s^{-1}. The mass of the ball is 46 g.

a Determine the mean accelerating force. [4]

b What mass, resting on the ball, would exert the same force as in **a**? [2]

2 The mass of a spacecraft is 70 kg. As the spacecraft takes off from the Moon, the upwards force on the spacecraft caused by the engines is 500 N. The gravitational field strength on the Moon is 1.6 N kg^{-1}.

Determine:

a the weight of the spacecraft on the Moon [2]

b the resultant force on the spacecraft [2]

c the acceleration of the spacecraft. [2]

3 A metal ball is dropped into a tall cylinder of oil. The ball initially accelerates but soon reaches a terminal velocity.

a By considering the forces on the metal ball bearing, explain why it first accelerates but then reaches terminal velocity. [3]

b Describe how you would show that the metal ball reaches terminal velocity. [3]

4 Determine the speed in m s^{-1} of an object that travels:

a 3 μm in 5 ms [2]

b 6 km in 3 Ms [2]

c 8 pm in 4 ns. [2]

5 Figure 3.18 shows a man who is just supporting the weight of a box. Two of the forces acting are shown in the diagram. According to Newton's third law, each of these forces is paired with **another** force.

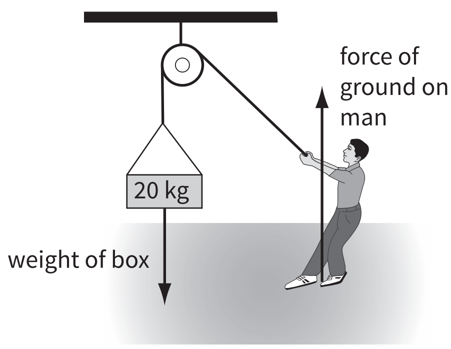


Figure 3.18 For End-of-chapter Question 5.

For **a** the weight of the box and **b** the force of the ground on the man, state:

i the body that the other force acts upon [2]

ii the direction of the other force [2]

iii the type of force involved. [2]

6 A car starts to move along a straight, level road. For the first 10 s, the driver maintains a constant acceleration of 1.5 m s^{-2}. The mass of the car is 1.1×10^3 kg.

a Calculate the driving force provided by the wheels, when:

i the force opposing motion is negligible [1]

ii the total force opposing the motion of the car is 600 N. [1]

b Calculate the distance travelled by the car in the first 10 s. [2]

7 Figure 3.19 shows the speed–time graphs for two falling balls.

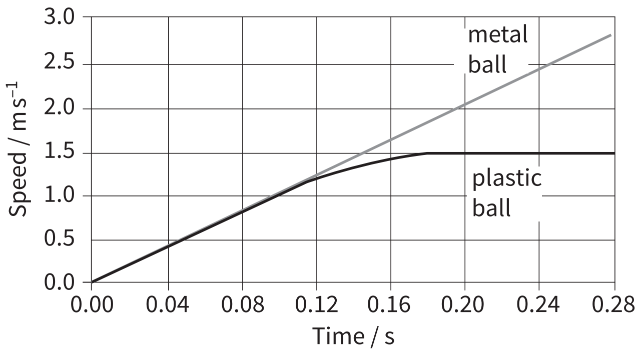


Figure 3.19 For End-of-chapter Question 7.

a Determine the terminal velocity of the plastic ball. [1]

b Both balls are of the same size and shape but the metal ball has a greater mass. Explain, in terms of Newton's laws of motion and the forces involved, why the plastic ball reaches a constant velocity but the metal ball does not. [3]

c Explain why both balls have the same initial acceleration. [2]

8 A car of mass 1200 kg accelerates from rest to a speed of 8.0 m s^{-1} in a time of 2.0 s.

a Calculate the forward driving force acting on the car while it is accelerating. Assume that, at low speeds, all frictional forces are negligible. [2]

b At high speeds the resistive frictional force F produced by air on a body moving with velocity v is given by the equation $F = bv^2$, where b is a constant.

i Derive the base units of force in the SI system. [1]

ii Determine the base units of b in the SI system. [1]

iii The car continues with the same forward driving force and accelerates until it reaches a top speed of 50 m s^{-1}. At this speed the resistive force is given by the equation $F = bv^2$. Determine the value of b for the car. [2]

iv Sketch a graph showing how the value of F varies with v over the range 0 to 50 m s^{-1}. Use your graph to describe what happens to the acceleration of the car during this time. [2]

9 a Explain what is meant by the **mass** of a body and the **weight** of a body. [3]

b State and explain **one** situation in which the weight of a body changes while its mass remains constant. [2]

c State the difference between the base units of mass and weight in the SI system. [2]

10 a State Newton's second law of motion. [2]

b When you jump from a wall on to the ground, it is advisable to bend your knees on landing.

i State how bending your knees affects the time it takes to stop when hitting the ground. [1]

ii Using Newton's second law of motion, explain why it is sensible to bend your knees. [2]

Chapter 4:
Forces – vectors and moments

Learning outcomes

You should be able to:

- add two or more coplanar forces
- resolve a force into perpendicular components
- define and apply the moment of a force and the torque of a couple
- apply the principle of moments
- state the conditions for a body to be in equilibrium

Sailing ahead

Force is a vector quantity. Sailors know a lot about the vector nature of forces. For example, they can sail 'into the wind'. The sails of a yacht can be angled to provide a component of force in the forward direction and the boat can then sail at almost 45° to the wind. The boat tends to 'heel over' and the crew sit on the side of the boat to provide a turning effect in the opposite direction (Figure 4.1).

Figure 4.1 Sailing into the wind.

Combining forces

You should recall that a vector quantity has both magnitude and direction. An object may have two or more forces acting on it and, since these are vectors, we must use vector addition (Chapter 1) to find their combined effect (their resultant).

There are several forces acting on the car (Figure 4.2) as it struggles up the steep hill. They are:

- its weight *W* (= *mg*)
- the contact force *N* of the road (its normal reaction)
- air resistance *D*
- the forward force *F* caused by friction between the car tyres and the road.

If we knew the magnitude and direction of each of these forces, we could work out their combined effect on the car. Will it accelerate up the hill? Or will it slide backwards down the hill?

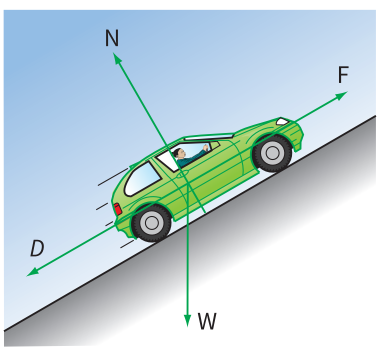


Figure 4.2 Four forces act on this car as it moves uphill.

The combined effect of several forces is known as the **resultant force**. To see how to work out the resultant of two or more forces, we will start with a relatively simple example.

Two forces in a straight line

We saw some examples in Chapter 3 of two forces acting in a straight line. For example, a falling tennis ball may be acted on by two forces: its weight mg, downwards, and air resistance D, upwards (Figure 4.3). The resultant force is then:

$$\text{resultant force} = mg - D = 1.0 - 0.2 = 0.8\,\text{N}$$

When adding two or more forces which act in a straight line, we have to take account of their directions. A force may be positive or negative; we adopt a **sign convention** to help us decide which is which.

If you apply a sign convention correctly, the sign of your final answer will tell you the direction of the resultant force (and hence acceleration).

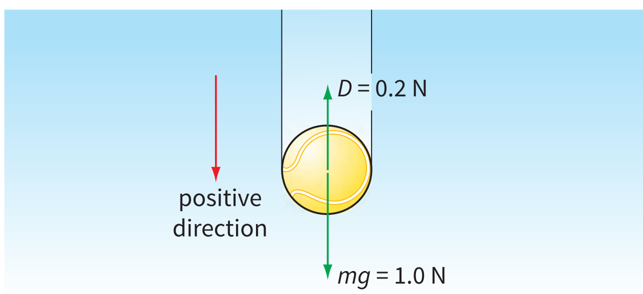


Figure 4.3 Two forces on a falling tennis ball.

Two forces at right angles

Figure 4.4 shows a shuttlecock falling on a windy day. There are two forces acting on the shuttlecock: its weight vertically downwards, and the horizontal push of the wind. (It helps if you draw the force arrows of different lengths, to show which force is greater.) We must add these two forces together to find the resultant force acting on the shuttlecock.

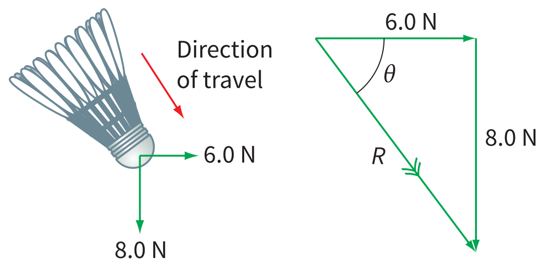


Figure 4.4 Two forces act on this shuttlecock as it travels through the air; the vector triangle shows how to find the resultant force.

We add the forces by drawing two arrows, end-to-end, as shown on the right of Figure 4.4.

- First, a horizontal arrow is drawn to represent the 6.0 N push of the wind.
- Next, starting from the end of this arrow, we draw a second arrow, downwards, representing the weight of 8.0 N.
- Now we draw a line from the start of the first arrow to the end of the second arrow. This arrow represents the resultant force R, in both magnitude and direction.

The arrows are added by drawing them end-to-end; the end of the first arrow is the start of the second arrow. Now we can find the resultant force either by scale drawing or by calculation. In this case, we have a 3–4–5 right-angled triangle, so calculation is simple:

$$R^2 = 6.0^2 + 8.0^2 = 36 + 64 = 100$$

$$R = 10\,\text{N}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8.0}{6.0} = \frac{4}{3}$$

$$\theta = \tan^{-1}\frac{4}{3} \approx 53°$$

So the resultant force is 10 N, at an angle of 53° below the horizontal. This is a reasonable answer; the weight is pulling the shuttlecock downwards and the wind is pushing it to the right. The angle is greater than 45° because the downward force is greater than the horizontal force.

If you draw a scale drawing be careful to:

- state the scale used
- draw a large diagram to reduce the uncertainty.

Three or more forces

The spider shown in Figure 4.5 is hanging by a thread. It is blown sideways by the wind. The diagram shows the three forces acting on it:

- weight acting downwards
- the tension in the thread
- the push of the wind.

The diagram also shows how these can be added together. In this case, we arrive at an interesting result. Arrows are drawn to represent each of the three forces, end-to-end. The end of the third arrow coincides with the start of the first arrow, so the three arrows form a closed triangle. This tells us that the resultant force R on the spider is zero, that is, $R = 0$. The closed triangle in Figure 4.5 is known as a **triangle of forces**.

So there is no resultant force. The forces on the spider balance each other out, and we say that the spider is in **equilibrium**. If the wind blew a little harder, there would be an unbalanced force on the spider, and it would move off to the right.

We can use this idea in two ways:

- If we work out the resultant force on an object and find that it is zero, this tells us that the object is in equilibrium.
- If we know that an object is in equilibrium, we know that the forces on it must add up to zero. We can use this to work out the values of one or more unknown forces.

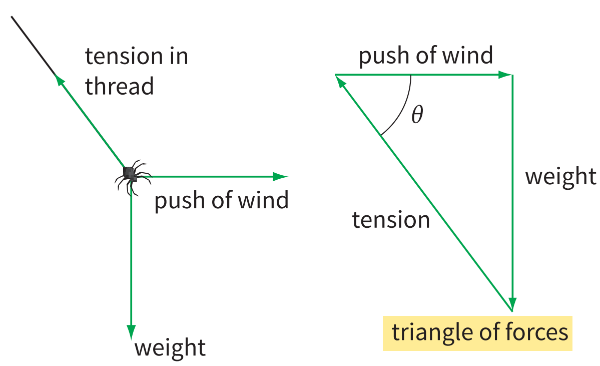


Figure 4.5 Blowing in the wind – this spider is hanging in equilibrium.

QUESTIONS

1 A parachutist weighs 1000 N. When she opens her parachute, it pulls upwards on her with a force of 2000 N.

   a Draw a diagram to show the forces acting on the parachutist.

   b Calculate the resultant force acting on her.

   c What effect will this force have on her?

2 The ship shown in Figure 4.6 is travelling at a constant velocity.

   a Is the ship in equilibrium (in other words, is the resultant force on the ship equal to zero)? How do you know?

   b What is the upthrust *U* of the water?

   c What is the drag *D* of the water?

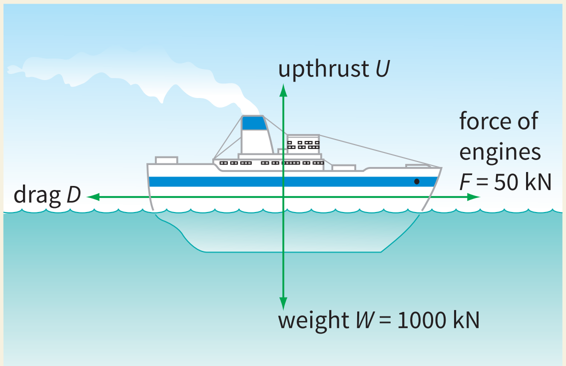


Figure 4.6 For Question 2. The force *D* is the frictional drag of the water on the boat. Like air resistance, drag is always in the opposite direction to the object's motion.

3 A stone is dropped into a fast-flowing stream. It does not fall vertically, because of the sideways push of the water (Figure 4.7).

   a Calculate the resultant force on the stone.

   b Is the stone in equilibrium?

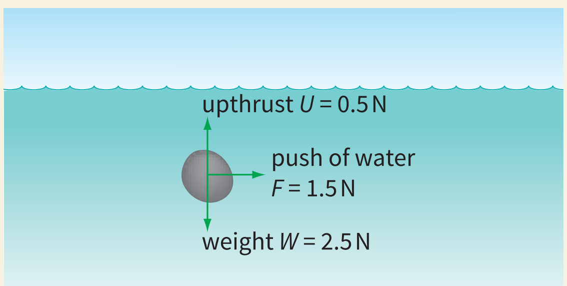


Figure 4.7 For Question 3.

Components of vectors

Look back to Figure 4.5. The spider is in equilibrium, even though three forces are acting on it. We can think of the tension in the thread as having two effects:

- it is pulling upwards, to counteract the downward effect of gravity
- it is pulling to the left, to counteract the effect of the wind.

We can say that this force has two effects or **components**: an upwards (vertical) component and a sideways (horizontal) component. It is often useful to split up a vector quantity into components like this, just as we did with velocity in Chapter 2. The components are in two directions at right angles to each other, often horizontal and vertical. The process is called **resolving** the vector. Then we can think about the effects of each component separately; we say that the perpendicular components are **independent** of one another. Because the two components are at 90° to each other, a change in one will have no effect on the other. Figure 4.8 shows how to resolve a force *F* into its horizontal and vertical components. These are:

horizontal component of *F*, $F_x = F\cos\theta$

vertical component of *F*, $F_y = F\sin\theta$

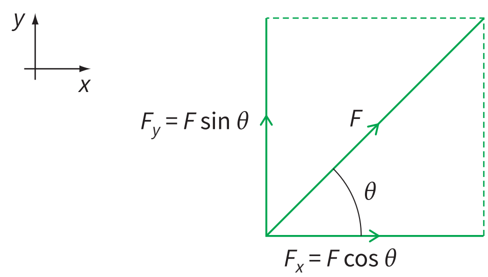


Figure 4.8 Resolving a vector into two components at right angles.

Making use of components

When the trolley shown in Figure 4.9 is released, it accelerates down the ramp. This happens because of the weight of the trolley. The weight acts vertically downwards, although this by itself does not determine the resulting motion. However, the weight has a component which acts down the slope. By calculating the component of the trolley's weight down the slope, we can determine its acceleration.

Figure 4.10 shows the forces acting on the trolley. To simplify the situation, we will assume there is no friction. The forces are:

- *W*, the weight of the trolley, which acts vertically downwards
- *N*, the contact force of the ramp, which acts at right angles to the ramp.

Figure 4.9 These students are investigating the acceleration of a trolley down a sloping ramp.

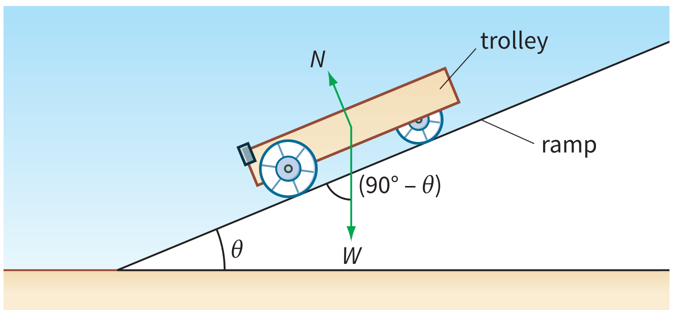


Figure 4.10 A force diagram for a trolley on a ramp.

You can see at once from the diagram that the forces cannot be balanced, since they do not act in the same straight line.

To find the component of W down the slope, we need to know the angle between W and the slope. The slope makes an angle θ with the horizontal, and from the diagram we can see that the angle between the weight and the ramp is $(90° - \theta)$. Using the rule for calculating the component of a vector given above, we have:

$$\text{component of } W \text{ down the slope} = W\cos(90° - \theta) = W\sin\theta$$

(It is helpful to recall that $\cos(90° - \theta) = \sin\theta$; you can see this from Figure 4.10.)

Does the contact force N help to accelerate the trolley down the ramp? To answer this, we must calculate its component down the slope. The angle between N and the slope is 90°. So:

$$\text{component of } N \text{ down the slope} = N\cos 90° = 0$$

The cosine of 90° is zero, and so N has no component down the slope. This shows why it is useful to think in terms of the components of forces; we don't know the value of N, but, since it has no effect down the slope, we can ignore it.

(There's no surprise about this result. The trolley runs down the slope because of the influence of its weight, not because it is pushed by the contact force N.)

Changing the slope

If the students in Figure 4.9 increase the slope of their ramp, the trolley will move down the ramp with greater acceleration. They have increased θ, and so the component of W down the slope will have increased.

Now we can work out the trolley's acceleration. If the trolley's mass is m, its weight is mg. So the force F making it accelerate down the slope is:

$$F = mg\sin\theta$$

Since from Newton's second law for constant mass we have $a = \frac{F}{m}$, the trolley's acceleration a is given by:

$$a = \frac{mg\sin\theta}{m} = g\sin\theta$$

We could have arrived at this result simply by saying that the trolley's acceleration would be the component of g down the slope (Figure 4.11). The steeper the slope, the greater the value of $\sin\theta$, and hence the greater the trolley's acceleration.

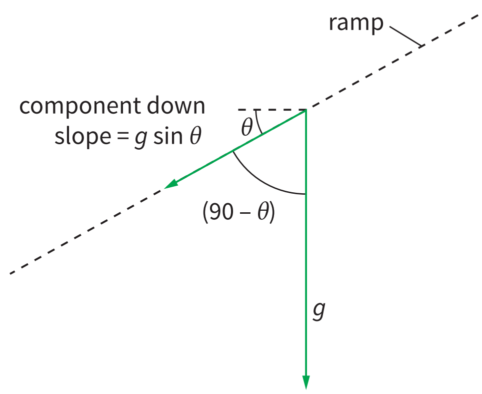


Figure 4.11 Resolving g down the ramp.

QUESTIONS

4 The person in Figure 4.12 is pulling a large box using a rope. Use the idea of components of a force to explain why they are more likely to get the box to move if the rope is horizontal (as in **a**) than if it is sloping upwards (as in **b**).

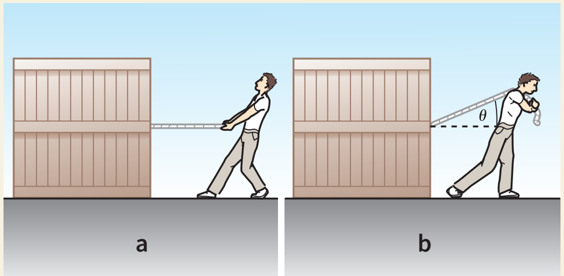


Figure 4.12 Why is it easier to move the box with the rope horizontal? See Question 4.

5 A crate is sliding down a slope. The weight of the crate is 500 N. The slope makes an angle of 30° with the horizontal.

- **a** Draw a diagram to show the situation. Include arrows to represent the weight of the crate and the contact force of the slope acting on the crate.
- **b** Calculate the component of the weight down the slope.
- **c** Explain why the contact force of the slope has no component down the slope.
- **d** What third force might act to oppose the motion? In which direction would it act?

Solving problems by resolving forces

A force can be resolved into two components at right angles to each other; these can then be treated independently of one another. This idea can be used to solve problems, as illustrated in Worked example 1.

QUESTION

6 A child of mass 40 kg is on a water slide. The slide slopes down at 25° to the horizontal. The acceleration of free fall is 9.81 m s^{-2}. Calculate the child's acceleration down the slope:

- **a** when there is no friction and the only force acting on the child is his weight
- **b** if a frictional force of 80 N acts up the slope.

WORKED EXAMPLE

1 A boy of mass 40 kg is on a waterslide which slopes at 30° to the horizontal. The frictional force up the slope is 120 N. Calculate the boy's acceleration down the slope. Take the acceleration of free fall g to be 9.81 m s^{-2}.

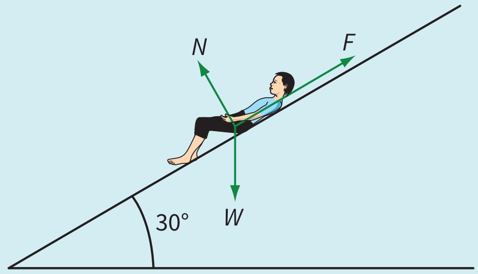


Figure 4.13 For Worked example 1.

Step 1 Draw a labelled diagram showing all the forces acting on the object of interest (Figure 4.13). This is known as a **free-body force diagram**. The forces are:

the boy's weight $W = 40 \times 9.81 = 392$ N

the frictional force up the slope $F = 120$ N

the contact force N at 90° to the slope.

Step 2 We are trying to find the resultant force on the boy which makes him accelerate down the slope. We resolve the forces down the slope, i.e. we find their components in that direction.

component of W down the slope $= 392 \times \cos 60°$
$= 196$ N

component of F down the slope $= -120$ N
(negative because F is directed up the slope)

component of N down the slope $= 0$
(because it is at 90° to the slope)

It is convenient that N has no component down the slope, since we do not know the value of N.

Step 3 Calculate the resultant force on the boy:

resultant force $= 196 - 120 = 76$ N

Step 4 Calculate his acceleration:

$$\text{acceleration} = \frac{\text{resultant force}}{\text{mass}} = \frac{76}{40} = 1.9\ \text{m s}^{-2}$$

So the boy's acceleration down the slope is 1.9 m s^{-2}. We could have arrived at the same result by resolving vertically and horizontally, but that would have led to two simultaneous equations from which we would have had to eliminate the unknown force N. It often helps to resolve forces at 90° to an unknown force.

Centre of gravity

We have weight because of the force of gravity of the Earth on us. Each part of our body – arms, legs, head, for example – experiences a force, caused by the force of gravity. However, it is much simpler to picture the overall effect of gravity as acting at a single point. This is our centre of gravity.

The centre of gravity of an object is defined as the point where all the weight of the object may be considered to act.

For a person standing upright, the centre of gravity is roughly in the middle of the body, behind the navel. For a sphere, it is at the centre. It is much easier to solve problems if we simply indicate an object's weight by a single force acting at the centre of gravity, rather than a large number of forces acting on each part of the object. Figure 4.14 illustrates this point. The athlete performs a complicated manoeuvre. However, we can see that his centre of gravity follows a smooth, parabolic path through the air, just like the paths of projectiles we discussed in Chapter 2.

Figure 4.14 The dots indicate the athlete's centre of gravity, which follows a smooth trajectory through the air. With his body curved like this, the athlete's centre of gravity is actually outside his body, just below the small of his back. At no time is the whole of his body above the bar.

BOX 4.1: Finding the centre of gravity

The centre of gravity of a thin sheet, or lamina, of cardboard or metal can be found by suspending it freely from two or three points (Figure 4.15).

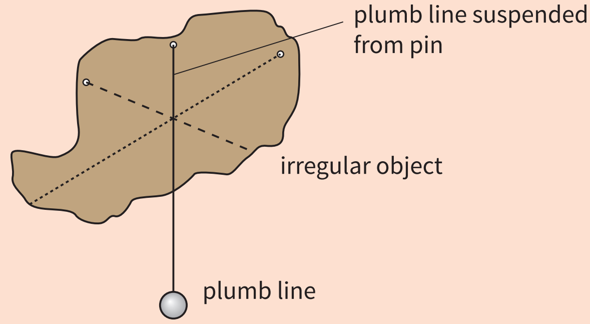


Figure 4.15 The centre of gravity is located at the intersection of the lines.

Small holes are made round the edge of the irregularly shaped object. A pin is put through one of the holes and held firmly in a clamp and stand so the object can swing freely. A length of string is attached to the pin. The other end of the string has a heavy mass attached to it. This arrangement is called a **plumb line**.

The object will stop swinging when its centre of gravity is vertically below the point of suspension. A line is drawn on the object along the vertical string of the plumb line. The centre of gravity must lie on this line. To find the position of the centre of gravity, the process is repeated with the object suspended from different holes. The centre of gravity will be at the point of intersection of the lines drawn on the object.

The turning effect of a force

Forces can make things accelerate. They can do something else as well: they can make an object turn round. We say that they can have a **turning effect**. Figure 4.16 shows how to use a spanner to turn a nut.

To maximise the turning effect of his force, the operator pulls close to the end of the spanner, as far as possible from the pivot (the centre of the nut) and at 90° to the spanner.

Figure 4.16 A mechanic turns a nut.

Moment of a force

The quantity which tells us about the turning effect of a force is its **moment**. The moment of a force depends on two quantities:

- the magnitude of the force (the bigger the force, the greater its moment)
- the perpendicular distance of the force from the pivot (the further the force acts from the pivot, the greater its moment).

The moment of a force is defined as follows:

> The moment of a force = force × perpendicular distance of the pivot from the line of action of the force.

Figure 4.17a shows these quantities. The force F_1 is pushing down on the lever, at a perpendicular distance x_1 from the pivot. The moment of the force F_1 about the pivot is then given by:

$$\text{moment} = \text{force} \times \text{distance from pivot}$$
$$= F_1 \times x_1$$

The unit of moment is the newton metre (N m). This is a unit which does not have a special name. You can also determine the moment of a force in N cm.

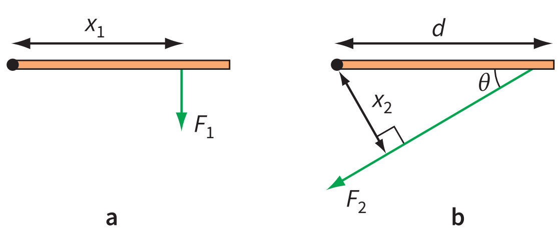


Figure 4.17 The quantities involved in calculating the moment of a force.

Figure 4.17b shows a slightly more complicated situation. F_2 is pushing at an angle θ to the lever, rather than at 90°. This makes it have less turning effect. There are two ways to calculate the moment of the force.

Method 1

Draw a perpendicular line from the pivot to the line of the force. Find the distance x_2. Calculate the moment of the force, $F_2 \times x_2$. From the right-angled triangle, we can see that:

$$x_2 = d \sin\theta$$

Hence:

$$\text{moment of force} = F_2 \times d \sin\theta = F_2 d \sin\theta$$

Method 2

Calculate the component of F_2 which is at 90° to the lever. This is $F_2 \sin\theta$. Multiply this by d.

$$\text{moment} = F_2 \sin\theta \times d$$

We get the same result as Method 1:

$$\text{moment of force} = F_2 d \sin\theta$$

Note that any force (such as the component $F_2 \cos\theta$) which passes through the pivot has no turning effect, because the distance from the pivot to the line of the force is zero.

Note also that we can calculate the moment of a force about any point, not just the pivot. However, in solving problems, it is often most convenient to take moments about the pivot as there is often an unknown force acting through the pivot (its contact force on the object).

Balanced or unbalanced?

We can use the idea of the moment of a force to solve two sorts of problem:

- We can check whether an object will remain balanced or start to rotate.
- We can calculate an unknown force or distance if we know that an object is balanced.

We can use the **principle of moments** to solve problems. The principle of moments states that:

> For any object that is in **equilibrium**, the sum of the clockwise moments about any point provided by the forces acting on the object equals the sum of the anticlockwise moments about that same point.

Note that, for an object to be in equilibrium, we also require that no resultant force acts on it. The Worked examples that follow illustrate how we can use these ideas to determine unknown forces.

WORKED EXAMPLES

2 Is the see-saw shown in Figure 4.18 in equilibrium (balanced), or will it start to rotate?

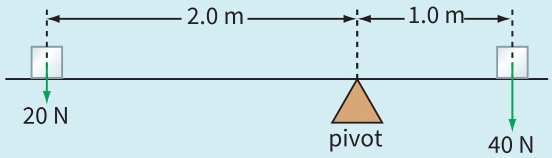


Figure 4.18 Will these forces make the see-saw rotate, or are their moments balanced?

The see-saw will remain balanced, because the 20 N force is twice as far from the pivot as the 40 N force.

To prove this, we need to think about each force individually. Which direction is each force trying to turn the see-saw, clockwise or anticlockwise? The 20 N force is tending to turn the see-saw anticlockwise, while the 40 N force is tending to turn it clockwise.

Step 1 Determine the anticlockwise moment:

moment of anticlockwise force = 20 × 2.0 = 40 N m

Step 2 Determine the clockwise moment:

moment of clockwise force = 40 × 1.0 = 40 N m

Step 3 We can see that:

clockwise moment = anticlockwise moment

So the see-saw is balanced and therefore does not rotate. The see-saw is in equilibrium.

3 The beam shown in Figure 4.19 is in equilibrium. Determine the force *X*.

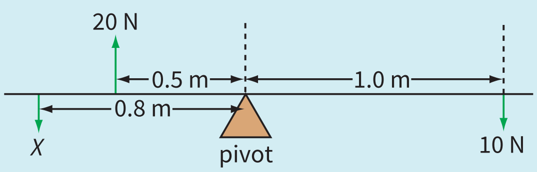


Figure 4.19 For Worked example 3.

The unknown force *X* is tending to turn the beam anticlockwise. The other two forces (10 N and 20 N) are tending to turn the beam clockwise. We will start by calculating their moments and adding them together.

Step 1 Determine the clockwise moments:

sum of moments of clockwise forces

= (10 × 1.0) + (20 × 0.5)

= 10 + 10 = 20 N m

Step 2 Determine the anticlockwise moment:

moment of anticlockwise force = $X \times 0.8$

Step 3 Since we know that the beam must be balanced, we can write:

sum of clockwise moments

= sum of anticlockwise moments

$$20 = X \times 0.8$$

$$X = \frac{20}{0.8} = 25\,\text{N}$$

So a force of 25 N at a distance of 0.8 m from the pivot will keep the beam still and prevent it from rotating (keep it balanced).

4 Figure 4.20 shows the internal structure of a human arm holding an object. The biceps are muscles attached to one of the bones of the forearm. These muscles provide an upward force.

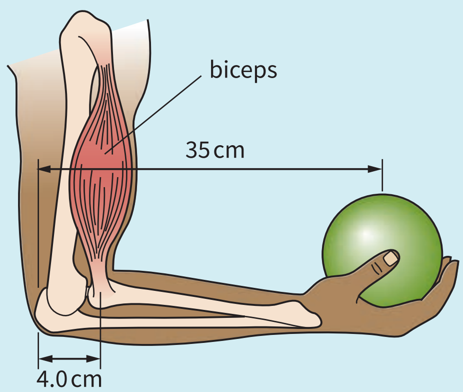


Figure 4.20 The human arm. For Worked example 4.

An object of weight 50 N is held in the hand with the forearm at right angles to the upper arm. Use the principle of moments to determine the muscular force *F* provided by the biceps, given the following data:

weight of forearm = 15 N

distance of biceps from elbow = 4.0 cm

distance of centre of gravity of forearm from elbow = 16 cm

distance of object in the hand from elbow = 35 cm

Step 1 There is a lot of information in this question. It is best to draw a simplified diagram of the forearm that shows all the forces and the relevant distances (Figure 4.21). All distances must be from the pivot, which in this case is the elbow.

WORKED EXAMPLES (continued)

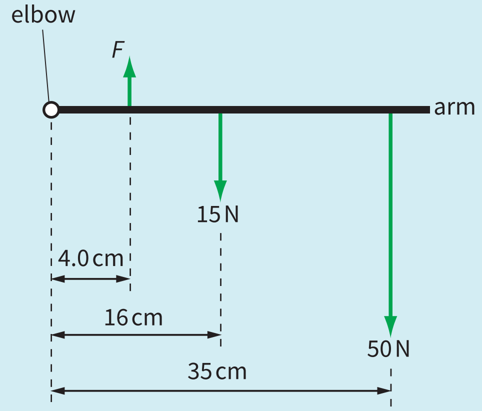


Figure 4.21 Simplified diagram showing forces on the forearm. For Worked example 4. Note that another force acts on the arm at the elbow; we do not know the size or direction of this force but we can ignore it by taking moments about the elbow.

Step 2 Determine the clockwise moments:

sum of moments of clockwise forces

$= (15 \times 0.16) + (50 \times 0.35)$

$= 19.9\ \text{N m}$

Step 3 Determine the anticlockwise moment:

moment of anticlockwise force $= F \times 0.04$

Step 4 Since the arm is in balance, according to the principle of moments we have:

sum of clockwise moments

= sum of anticlockwise moments

$19.9 = 0.04\,F$

$$F = \frac{19.9}{0.04} = 497.5\ \text{N} \approx 500\ \text{N}$$

The biceps provide a force of 500 N – a force large enough to lift 500 apples!

QUESTIONS

7 A wheelbarrow is loaded as shown in Figure 4.22.

   a Calculate the force that the gardener needs to exert to hold the wheelbarrow's legs off the ground.

   b Calculate the force exerted by the ground on the legs of the wheelbarrow (taken both together) when the gardener is not holding the handles.

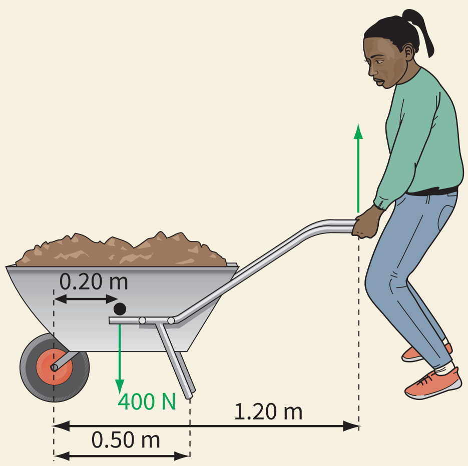


Figure 4.22 For Question 7.

8 A traditional pair of scales uses sliding masses of 10 g and 100 g to achieve a balance. A diagram of the arrangement is shown in Figure 4.23. The bar itself is supported with its centre of gravity at the pivot.

   a Calculate the value of the mass M, attached at X.

   b State **one** advantage of this method of measuring mass.

   c Determine the upward force of the pivot on the bar.

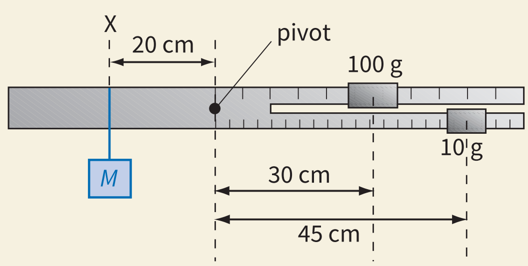


Figure 4.23 For Question 8.

9 Figure 4.24 shows a beam with four forces acting on it

   a For each force, calculate the moment of the force about point P.

   b State whether each moment is clockwise or anticlockwise.

   c State whether or not the moments of the forces are balanced.

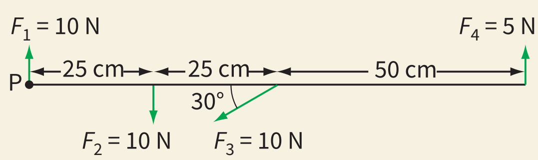


Figure 4.24 For Question 9.

The torque of a couple

Figure 4.25 shows the forces needed to turn a car's steering wheel. The two forces balance up and down (15 N up and 15 N down), so the wheel will not move up, down or sideways. However, the wheel is not in equilibrium. The pair of forces will cause it to rotate.

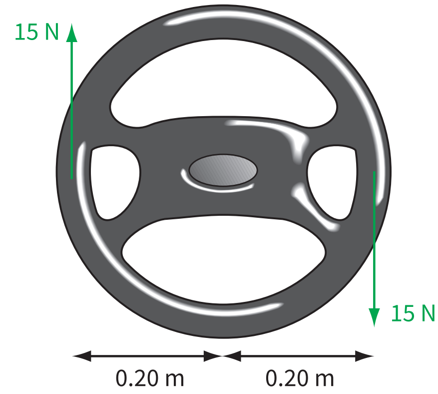


Figure 4.25 Two forces act on this steering wheel to make it turn.

A pair of forces like that in Figure 4.25 is known as a **couple**. A couple has a turning effect, but does not cause an object to accelerate. To form a couple, the two forces must be:

- equal in magnitude
- parallel, but opposite in direction
- separated by a distance *d*.

The turning effect or moment of a couple is known as its **torque**. We can calculate the torque of the couple in Figure 4.25 by adding the moments of each force about the centre of the wheel:

$$\text{torque of couple} = (15 \times 0.20) + (15 \times 0.20)$$
$$= 6.0\,\text{N m}$$

We could have found the same result by multiplying one of the forces by the perpendicular distance between them:

$$\text{torque of a couple} = 15 \times 0.4 = 6.0\,\text{N m}$$

The torque of a couple is defined as follows:

torque of a couple = one of the forces × perpendicular distance between the forces

QUESTION

10 The driving wheel of a car travelling at a constant velocity has a torque of 137 N m applied to it by the axle that drives the car (Figure 4.26). The radius of the tyre is 0.18 m. Calculate the driving force provided by this wheel.

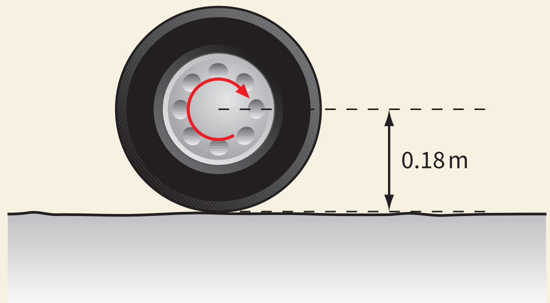


Figure 4.26 For Question 10.

Pure turning effect

When we calculate the moment of a single force, the result depends on the point or pivot about which the moment acts. The further the force is from the pivot, the greater the moment. A couple is different; the moment of a couple does not depend on the point about which it acts, only on the perpendicular distance between the two forces. A single force acting on an object will tend to make the object accelerate (unless there is another force to balance it). A couple, however, is a pair of equal and opposite forces, so it will not make the object accelerate. This means we can think of a couple as a pure 'turning effect', the size of which is given by its torque.

For an object to be in equilibrium, two conditions must be met at the same time:

- The resultant force acting on the object is zero.
- The resultant moment is zero.

Summary

- Forces are vector quantities that can be added by means of a vector triangle. Their resultant can be determined using trigonometry or by scale drawing.
- Vectors such as forces can be resolved into components. Components at right angles to one another can be treated independently of one another. For a force F at an angle θ to the x-direction, the components are:

  x-direction: $F\cos\theta$

  y-direction: $F\sin\theta$
- The moment of a force = force × perpendicular distance of the pivot from the line of action of the force.
- The principle of moments states that, for any object that is in equilibrium, the sum of the clockwise moments about any point provided by the forces acting on the object equals the sum of the anticlockwise moments about that same point.
- A couple is a pair of equal, parallel but opposite forces whose effect is to produce a turning effect on a body without giving it linear acceleration.

  torque of a couple = one of the forces × perpendicular distance between the forces
- For an object to be in equilibrium, the resultant force acting on the object must be zero and the resultant moment must be zero.

End-of-chapter questions

1 A ship is pulled at a constant speed by two small boats, A and B, as shown in Figure 4.27. The engine of the ship does not produce any force.

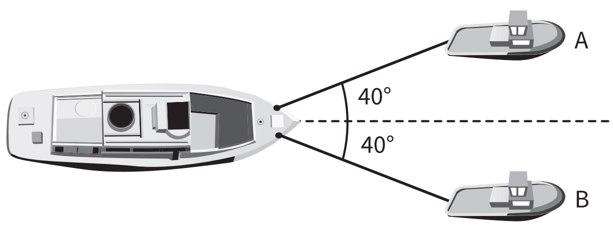


Figure 4.27 For End-of-chapter Question 1.

The tension in each cable between A and B and the ship is 4000 N.

a Draw a free-body diagram showing the three horizontal forces acting on the ship. [2]

b Draw a vector diagram to scale showing these three forces and use your diagram to find the value of the drag force on the ship. [2]

2 A block of mass 1.5 kg is at rest on a rough surface which is inclined at 20° to the horizontal as shown in Figure 4.28.

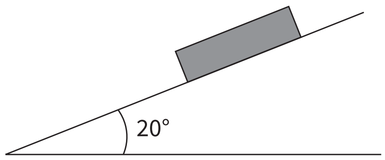


Figure 4.28 For End-of-chapter Question 2.

- **a** Draw a free-body diagram showing the three forces acting on the block. [2]
- **b** Calculate the component of the weight that acts down the slope. [2]
- **c** Use your answer to **b** to determine the force of friction that acts on the block. [2]
- **d** Determine the normal contact force between the block and the surface. [3]

3 The free-body diagram (Figure 4.29) shows three forces that act on a stone hanging at rest from two strings.

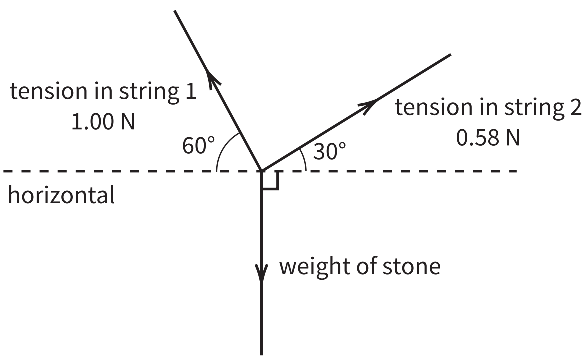


Figure 4.29 For End-of-chapter Question 3.

- **a** Calculate the horizontal component of the tension in each string. Why should these two components be equal in magnitude? [5]
- **b** Calculate the vertical component of the tension in each string. [4]
- **c** Use your answer to **b** to calculate the weight of the stone. [2]
- **d** Draw a vector diagram of the forces on the stone. This should be a triangle of forces. [1]
- **e** Use your diagram in **d** to calculate the weight of the stone. [2]

4 The force *F* shown in Figure 4.30 has a moment of 40 N m about the pivot. Calculate the magnitude of the force *F*. [4]

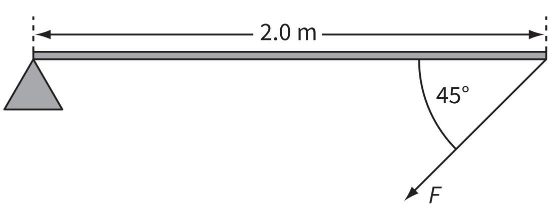


Figure 4.30 For End-of-chapter Question 4.

5 The asymmetric bar shown in Figure 4.31 has a weight of 7.6 N and a centre of gravity that is 0.040 m from the wider end, on which there is a load of 3.3 N. It is pivoted a distance of 0.060 m from its centre of gravity. Calculate the force P that is needed at the far end of the bar in order to maintain equilibrium. [4]

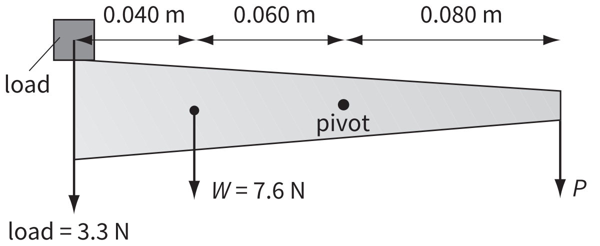


Figure 4.31 For End-of-chapter Question 5.

6 **a** Explain what is meant by:

  i a couple [1]

  ii torque. [2]

b The engine of a car produces a torque of 200 N m on the axle of the wheel in contact with the road. The car travels at a constant velocity towards the right (Figure 4.32).

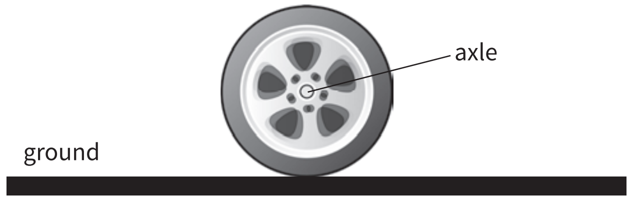


Figure 4.32 For End-of-chapter Question 6.

  i Copy Figure 4.32 and show the direction of rotation of the wheel, and the horizontal component of the force that the road exerts on the wheel. [2]

  ii State the resultant torque on the wheel. Explain your answer. [2]

  iii The diameter of the car wheel is 0.58 m. Determine the value of the horizontal component of the force of the road on the wheel. [1]

7 **a** Explain what is meant by the **centre of gravity** of an object. [2]

b A flagpole of mass 25 kg is held in a horizontal position by a cable as shown in Figure 4.33. The centre of gravity of the flagpole is at a distance of 1.5 m from the fixed end.

  i Write an equation to represent taking moments about the left-hand end of the flagpole. Use your equation to find the tension T in the cable. [4]

  ii Determine the vertical component of the force at the left-hand end of the flagpole. [2]

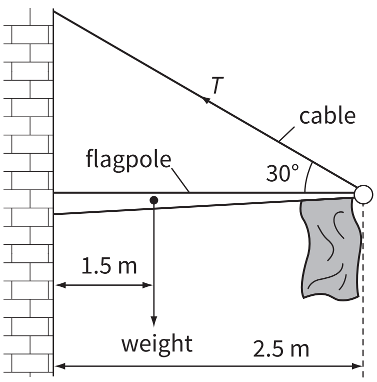


Figure 4.33 For End-of-chapter Question 7.

8 **a** State the **two** conditions necessary for an object to be in equilibrium. [2]

b A metal rod of length 90 cm has a disc of radius 24 cm fixed rigidly at its centre, as shown in Figure 4.34 The assembly is pivoted at its centre.

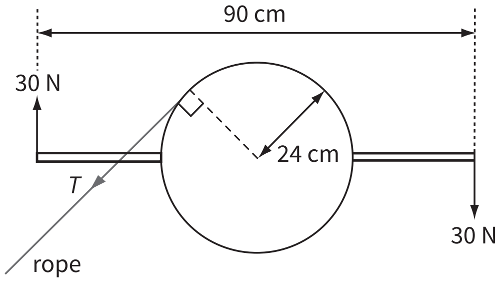


Figure 4.34 For End-of-chapter Question 8.

Two forces, each of magnitude 30 N, are applied normal to the rod at each end so as to produce a turning effect on the rod. A rope is attached to the edge of the disc to prevent rotation. Calculate:

i the torque of the couple produced by the 30 N forces [1]

ii the tension *T* in the rope. [3]

9 **a** Explain what is meant by the **torque of a couple**. [2]

b Three strings, A, B and C, are attached to a circular ring, as shown in Figure 4.35.

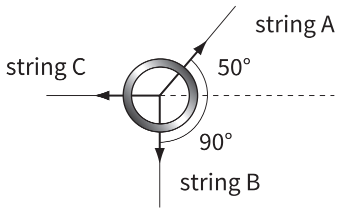


Figure 4.35 For End-of-chapter Question 9.

The strings and the ring all lie on a smooth horizontal surface and are at rest. The tension in string A is 8.0 N. Calculate the tension in strings B and C. [4]

10 Figure 4.36 shows a picture hanging symmetrically by two cords from a nail fixed to a wall. The picture is in equilibrium.

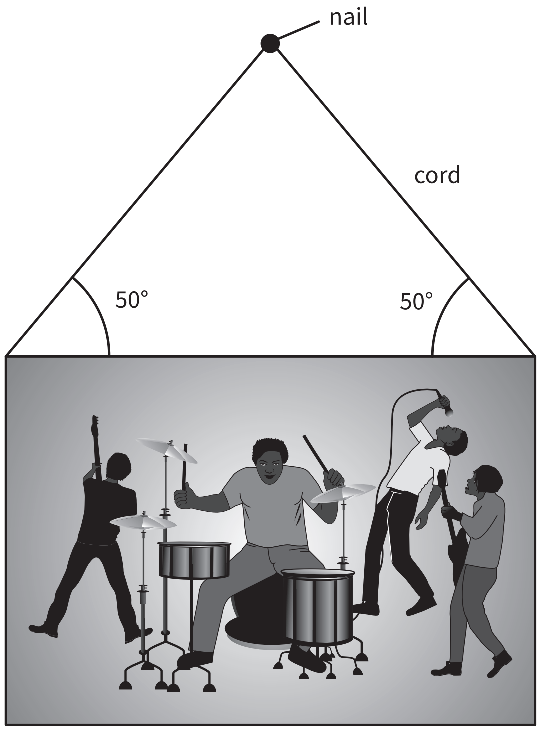


Figure 4.36 For End-of-chapter Question 10.

a Explain what is meant by **equilibrium**. [2]

b Draw a vector diagram to represent the three forces acting on the picture in the vertical plane. Label each force clearly with its name and show the direction of each force with an arrow. [2]

c The tension in the cord is 45 N and the angle that each end of the cord makes with the horizontal is 50°. Calculate:

i the vertical component of the tension in the cord [1]

ii the weight of the picture. [1]

Chapter 5: Work, energy and power

Learning outcomes

You should be able to:

- give examples of conversions of energy between different forms
- understand and use the concept of work
- apply the principle of conservation of energy to simple examples involving energy in different forms
- derive and use the formulae for kinetic energy and potential energy
- define and use the equation for power

The idea of energy

The Industrial Revolution started in the late 18th century in the British Isles. Today, many other countries are undergoing the process of industrialisation (Figure 5.1). Industrialisation began as engineers developed new machines which were capable of doing the work of hundreds of craftsmen and labourers. At first, they made use of the traditional techniques of water power and wind power. Water stored behind a dam was used to turn a wheel, which turned many machines. By developing new mechanisms, the designers tried to extract as much as possible of the energy stored in the water. Steam engines were developed, initially for pumping water out of mines. Steam engines use a fuel such as coal; there is much more energy stored in 1 kg of coal than in 1 kg of water held behind a dam. Steam engines soon powered the looms of the textile mills, and the British industry came to dominate world trade in textiles.

Nowadays, most factories and mills rely on electrical power, generated by burning coal or gas at a power station. The fuel is burnt to release its store of energy. High-pressure steam is generated, and this turns a turbine which turns a generator. Even in the most efficient coal-fired power station, only about 40% of the energy from the fuel is transferred to the electrical energy that the station supplies to the grid.

Engineers strove to develop machines which made the most efficient use of the energy supplied to them. At the same time, scientists were working out the basic ideas of energy transfer and energy transformations. The idea of energy itself had to be developed; it was not obvious at first that heat, light, electrical energy and so on could all be thought of as being, in some way, forms of the same thing. In fact, steam engines had been in use for 150 years before it was realised that their energy came from the heat supplied to them from their fuel.

The earliest steam engines had very low efficiencies – many converted less than 1% of the energy supplied to them into useful work. The understanding of the relationship between work and energy led to many ingenious ways of making the most of the energy supplied by fuel.

This improvement in energy efficiency has led to the design of modern engines such as the jet engines which have made long-distance air travel a commercial possibility (Figure 5.2).

Figure 5.2 The jet engines of this aircraft are designed to make efficient use of their fuel. If they were less efficient, their thrust might only be sufficient to lift the empty aircraft, and the passengers would have to be left behind.

Figure 5.1 Anshan steel works, China.

Doing work, transferring energy

The weight-lifter shown in Figure 5.3 has powerful muscles. They can provide the force needed to lift a large weight above her head – about 2 m above the ground. The force exerted by the weight-lifter transfers energy from her to the weights. We know that the weights have gained energy because, when the athlete releases them, they come crashing down to the ground.

Figure 5.3 It is hard work being a weight-lifter.

As the athlete lifts the weights and transfers energy to them, we say that her lifting force is doing work. 'Doing work' is a way of transferring energy from one object to another. In fact, if you want to know the scientific meaning of the word 'energy', we have to say it is 'that which is transferred when a force moves through a distance'. So work and energy are two closely linked concepts.

In physics, we often use an everyday word but with a special meaning. **Work** is an example of this. Table 5.1 describes some situations which illustrate the meaning of **doing work** in physics.

It is important to appreciate that our bodies sometimes mislead us. If you hold a heavy weight above your head for some time, your muscles will get tired. However,

Doing work	Not doing work
Pushing a car to start it moving: your force transfers energy to the car. The car's kinetic energy (i.e. 'movement energy') increases.	Pushing a car but it does not budge: no energy is transferred, because your force does not move it. The car's kinetic energy does not change.
Lifting weights: you are doing work as the weights move upwards. The gravitational potential energy of the weights increases.	Holding weights above your head: you are not doing work on the weights (even though you may find it tiring) because the force you apply is not moving them. The gravitational potential energy of the weights is not changing.
A falling stone: the force of gravity is doing work. The stone's kinetic energy is increasing.	The Moon orbiting the Earth: the force of gravity is not doing work. The Moon's kinetic energy is not changing.
Writing an essay: you are doing work because you need a force to move your pen across the page, or to press the keys on the keyboard.	Reading an essay: this may seem like 'hard work', but no force is involved, so you are not doing any work.

Table 5.1 The meaning of 'doing work' in physics.

you are not doing any work **on the weights**, because you are not transferring energy to the weights once they are above your head. Your muscles get tired because they are constantly relaxing and contracting, and this uses energy, but none of the energy is being transferred to the weights.

Calculating work done

Because **doing work** defines what we mean by **energy**, we start this chapter by considering how to calculate **work done**. There is no doubt that you do work if you push a car along the road. A force transfers energy from you to the car. But how much work do you do? Figure 5.4 shows the two factors involved:

- the size of the force F – the bigger the force, the greater the amount of work you do
- the distance s you push the car – the further you push it, the greater the amount of work done.

So, the bigger the force, and the further it moves, the greater the amount of work done.

The **work done** by a force is defined as the product of the force and the distance moved in the direction of the force:

$W = F \times s$

where s is the distance moved in the direction of the force.

In the example shown in Figure 5.4, $F = 300\,\mathrm{N}$ and $s = 5.0\,\mathrm{m}$, so:

work done $W = F \times s = 300 \times 5.0 = 1500\,\mathrm{J}$

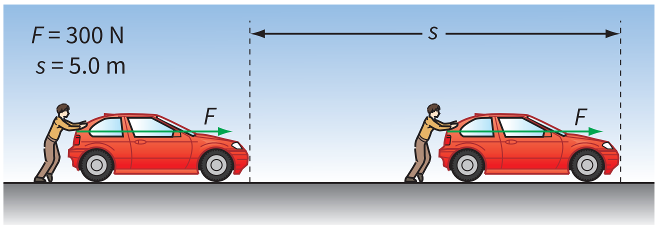


Figure 5.4 You have to do work to start the car moving.

Energy transferred

Doing work is a way of transferring energy. For both energy and work the correct SI unit is the joule (J). The amount of work done, calculated using $W = F \times s$, shows the amount of energy transferred:

work done = energy transferred

Newtons, metres and joules

From the equation $W = F \times s$ we can see how the unit of force (the newton), the unit of distance (the metre) and the unit of work or energy (the joule) are related.

1 joule = 1 newton × 1 metre

1 J = 1 N m

The joule is defined as the amount of work done when a force of 1 newton moves a distance of 1 metre in the direction of the force. Since **work done = energy transferred**, it follows that a joule is also the amount of energy transferred when a force of 1 newton moves a distance of 1 metre in the direction of the force.

QUESTIONS

1. In each of the following examples, explain whether or not any work is done by the force mentioned.
   - **a** You pull a heavy sack along rough ground.
   - **b** The force of gravity pulls you downwards when you fall off a wall.
   - **c** The tension in a string pulls on a stone when you whirl it around in a circle at a steady speed.
   - **d** The contact force of the bedroom floor stops you from falling into the room below.
2. A man of mass 70 kg climbs stairs of vertical height 2.5 m. Calculate the work done against the force of gravity. (Take $g = 9.81\,\mathrm{m\,s^{-2}}$.)
3. A stone of weight 10 N falls from the top of a 250 m high cliff.
   - **a** Calculate how much work is done by the force of gravity in pulling the stone to the foot of the cliff.
   - **b** How much energy is transferred to the stone?

Force, distance and direction

It is important to appreciate that, for a force to do work, there must be movement in the direction of the force. Both the force F and the distance s moved **in the direction of the force** are vector quantities, so you should know that their directions are likely to be important. To illustrate this, we will consider three examples involving gravity (Figure 5.5). In the equation for work done, $W = F \times s$, the distance moved s is thus the displacement in the direction of the force.

Suppose that the force F moves through a distance s which is at an angle θ to F, as shown in Figure 5.6. To determine the work done by the force, it is simplest to determine the component of F in the direction of s. This component is $F\cos\theta$, and so we have:

work done = $(F\cos\theta) \times s$

or simply:

work done = $Fs\cos\theta$

Worked example 1 shows how to use this.

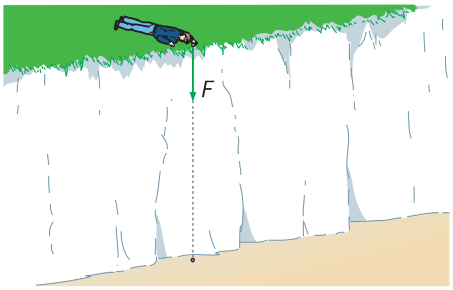
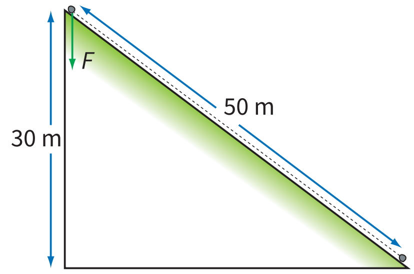
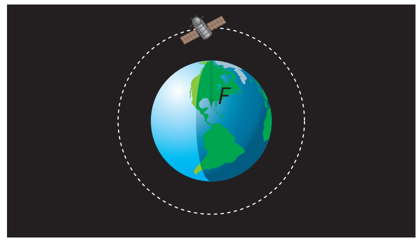
Doing work		Not doing work
		
1 You drop a stone weighing 5.0 N from the top of a 50 m high cliff. What is the work done by the force of gravity? force on stone F = pull of gravity = weight of stone = 5.0 N vertically downwards	**2** A stone weighing 5.0 N rolls 50 m down a slope. What is the work done by the force of gravity? force on stone F = pull of gravity = weight of stone = 5.0 N vertically downwards	**3** A satellite orbits the Earth at a constant height and at a constant speed. The weight of the satellite at this height is 500 N. What is the work done by the force of gravity? force on satellite F = pull of gravity = weight of satellite = 500 N towards centre of Earth
Distance moved by stone is s = 50 m vertically downwards.	Distance moved by stone down slope is 50 m, but distance moved in direction of force is 30 m.	Distance moved by satellite towards centre of Earth (i.e. in the direction of force) is s = 0.
Since F and s are in the same direction, there is no problem: work done = $F \times s$ = 5.0 × 50 = 250 J	The work done by the force of gravity is: work done = 5.0 × 30 = 150 J	The satellite remains at a constant distance from the Earth. It does not move in the direction of F. The work done by the Earth's pull on the satellite is zero because F = 500 N but s = 0: work done = 500 × 0 = 0 J

Figure 5.5 Three examples involving gravity.

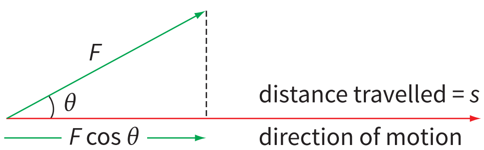


Figure 5.6 The work done by a force depends on the angle between the force and the distance it moves.

WORKED EXAMPLE

1 A man pulls a box along horizontal ground using a rope (Figure 5.7). The force provided by the rope is 200 N, at an angle of 30° to the horizontal. Calculate the work done if the box moves 5.0 m along the ground.

Step 1 Calculate the component of the force in the direction in which the box moves. This is the horizontal component of the force:

horizontal component of force = 200 cos 30° ≈ 173 N

Hint: *$F \cos \theta$ is the component of the force at an angle θ to the direction of motion.*

Step 2 Now calculate the work done:

work done = force × distance moved = 173 × 5.0 = 865 J

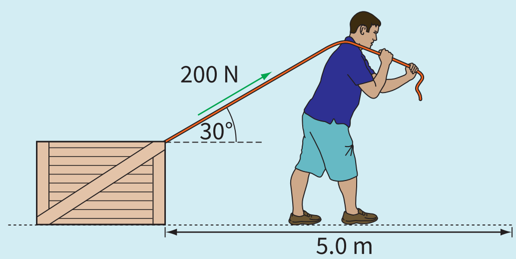


Figure 5.7 For Worked example 1.

Hint: *Note that we could have used the equation work done = $Fs \cos \theta$ to combine the two steps into one.*

A gas doing work

Gases exert pressure on the walls of their container. If a gas expands, the walls are pushed outwards – the gas has done work on its surroundings. In a steam engine, expanding steam pushes a piston to turn the engine, and in a car engine, the exploding mixture of fuel and air does the same thing, so this is an important situation.

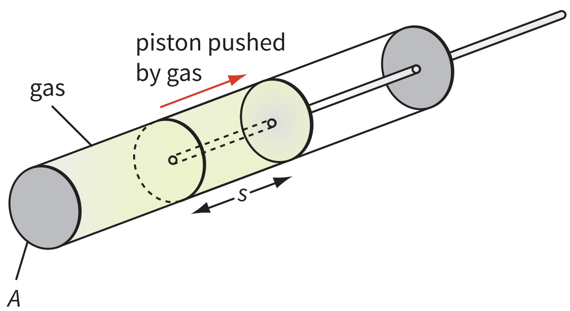


Figure 5.8 When a gas expands, it does work on its surroundings.

Figure 5.8 shows a gas at pressure p inside a cylinder of cross-sectional area A. The cylinder is closed by a moveable piston. The gas pushes the piston a distance s. If we know the force F exerted by the gas on the piston, we can deduce an expression for the amount of work done by the gas.

From the definition of pressure (pressure = $\frac{\text{force}}{\text{area}}$), the force exerted by the gas on the piston is given by:

force = pressure × area

$$F = p \times A$$

and the work done is force × displacement:

$$W = p \times A \times s$$

But the quantity $A \times s$ is the **increase** in volume of the gas; that is, the shaded volume in Figure 5.8. We call this ΔV, where the Δ indicates that it is a **change** in V. Hence the work done by the gas in expanding is:

$$W = p\Delta V$$

Notice that we are assuming that the pressure p does not change as the gas expands. This will be true if the gas is expanding against the pressure of the atmosphere, which changes only very slowly.

QUESTIONS

4 The crane shown in Figure 5.9 lifts its 500 N load to the top of the building from A to B. Distances are as shown on the diagram. Calculate how much work is done by the crane.

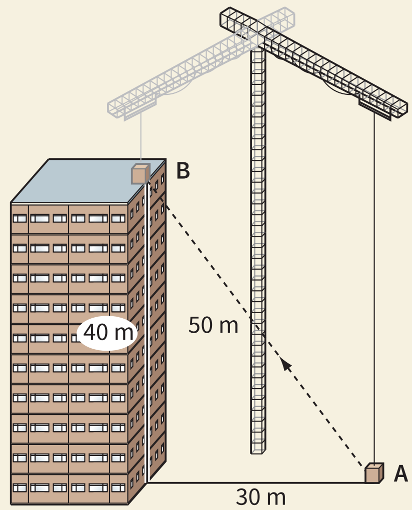


Figure 5.9 For Question 4. The dotted line shows the track of the load as it is lifted by the crane.

5 Figure 5.10 shows the forces acting on a box which is being pushed up a slope. Calculate the work done by each force if the box moves 0.50 m up the slope.

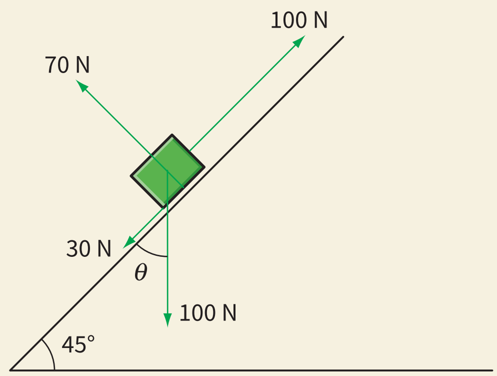


Figure 5.10 For Question 5.

6 When you blow up a balloon, the expanding balloon pushes aside the atmosphere. How much work is done against the atmosphere in blowing up a balloon to a volume of 2 litres ($0.002\ m^3$)? (Atmospheric pressure = $1.0 \times 10^5\ N\ m^{-2}$.)

Gravitational potential energy

If you lift a heavy object, you do work. You are providing an upward force to overcome the downward force of gravity on the object. The force moves the object upwards, so the force is doing work.

In this way, energy is transferred from you to the object. You lose energy, and the object gains energy. We say that the **gravitational potential energy** E_p of the object has increased. Worked example 2 shows how to calculate a change in gravitational potential energy – or g.p.e. for short.

WORKED EXAMPLE

2 A weight-lifter raises weights with a mass of 200 kg from the ground to a height of 1.5 m. Calculate how much work he does. By how much does the g.p.e. of the weights increase?

Step 1 As shown in Figure 5.11, the downward force on the weights is their weight, $W = mg$. An equal, upward force F is required to lift them.

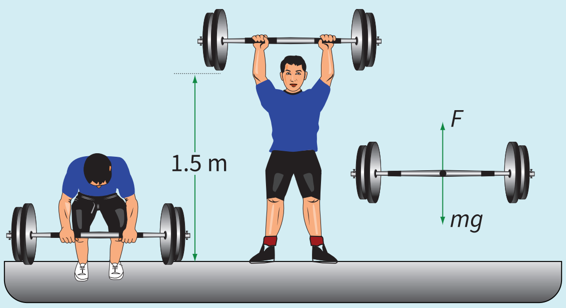


Figure 5.11 For Worked example 2.

$W = F = mg = 200 \times 9.81 = 1962\,\text{N}$

Hint: *It helps to draw a diagram of the situation.*

Step 2 Now we can calculate the work done by the force F:

work done = force × distance moved

$= 1962 \times 1.5 \approx 2940\,\text{J}$

Note that the distance moved is in the same direction as the force. So the work done on the weights is about 2940 J. This is also the value of the increase in their g.p.e.

An equation for gravitational potential energy

The change in the gravitational potential energy (g.p.e.) of an object, E_p, depends on the change in its height, h. We can calculate E_p using this equation:

change in g.p.e. = weight × change in height

$$E_p = (mg) \times h$$

or simply

$$E_p = mgh$$

It should be clear where this equation comes from. The force needed to lift an object is equal to its weight mg, where m is the mass of the object and g is the acceleration of free fall or the gravitational field strength on the Earth's surface. The work done by this force is given by force × distance moved, or weight × change in height. You might feel that it takes a force greater than the weight of the object being raised to lift it upwards, but this is not so. Provided the force is equal to the weight, the object will move upwards at a steady speed.

Note that h stands for the vertical height through which the object moves. Note also that we can only use the equation $E_p = mgh$ for relatively small changes in height. It would not work, for example, in the case of a satellite orbiting the Earth. Satellites orbit at a height of at least 200 km and g has a smaller value at this height.

Other forms of potential energy

Potential energy is the energy an object has because of its position or shape. So, for example, an object's gravitational potential energy changes when it moves through a gravitational field. (There is much more about gravitational fields in Chapter 18.)

We can identify other forms of potential energy. An electrically charged object has electric potential energy when it is placed in an electric field (see Chapter 8). An object may have elastic potential energy when it is stretched, squashed or twisted – if it is released it goes back to its original shape (see Chapter 7).

QUESTIONS

7 Calculate how much gravitational potential energy is gained if you climb a flight of stairs. Assume that you have a mass of 52 kg and that the height you lift yourself is 2.5 m.

8 A climber of mass 100 kg (including the equipment she is carrying) ascends from sea level to the top of a mountain 5500 m high. Calculate the change in her gravitational potential energy.

9 **a** A toy car works by means of a stretched rubber band. What form of potential energy does the car store when the band is stretched?

b A bar magnet is lying with its north pole next to the south pole of another bar magnet. A student pulls them apart. Why do we say that the magnets' potential energy has increased? Where has this energy come from?

Kinetic energy

As well as lifting an object, a force can make it accelerate. Again, work is done by the force and energy is transferred to the object. In this case, we say that it has gained kinetic energy, E_k. The faster an object is moving, the greater its kinetic energy (k.e.).

For an object of mass m travelling at a speed v, we have:

$$\text{kinetic energy} = \tfrac{1}{2} \times \text{mass} \times \text{speed}^2$$

$$E_k = \tfrac{1}{2}mv^2$$

Deriving the formula for kinetic energy

The equation for k.e., $E_k = \frac{1}{2}mv^2$, is related to one of the equations of motion. We imagine a car being accelerated from rest ($u = 0$) to velocity v. To give it acceleration a, it is pushed by a force F for a distance s. Since $u = 0$, we can write the equation $v^2 = u^2 + 2as$ as:

$$v^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$ gives:

$$\tfrac{1}{2}mv^2 = mas$$

Now, ma is the force F accelerating the car, and mas is the force × the distance it moves, that is, the work done by the force. So we have:

$$\tfrac{1}{2}mv^2 = \text{work done by force } F$$

This is the energy transferred to the car, and hence its kinetic energy.

WORKED EXAMPLE

3 Calculate the increase in kinetic energy of a car of mass 800 kg when it accelerates from 20 m s^{-1} to 30 m s^{-1}.

Step 1 Calculate the initial k.e. of the car:

$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 800 \times (20)^2 = 160\,000$ J

$= 160$ kJ

Step 2 Calculate the final k.e. of the car:

$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 800 \times (30)^2 = 360\,000$ J

$= 360$ kJ

Step 3 Calculate the change in the car's k.e.:

change in k.e. = 360 − 160 = 200 kJ

Hint: *Take care! You can't calculate the change in k.e. by squaring the change in speed. In this example, the change in speed is 10 m s^{-1}, and this would give an incorrect value for the change in k.e.*

QUESTIONS

10 Which has more k.e., a car of mass 500 kg travelling at 15 m s^{-1} or a motorcycle of mass 250 kg travelling at 30 m s^{-1}?

11 Calculate the change in kinetic energy of a ball of mass 200 g when it bounces. Assume that it hits the ground with a speed of 15.8 m s^{-1} and leaves it at 12.2 m s^{-1}.

g.p.e.–k.e. transformations

A motor drags the roller-coaster car to the top of the first hill. The car runs down the other side, picking up speed as it goes (see Figure 5.12). It is moving just fast enough to reach the top of the second hill, slightly lower than the first. It accelerates downhill again. Everybody screams!

The motor provides a force to pull the roller-coaster car to the top of the hill. It transfers energy to the car. But where is this energy when the car is waiting at the top of the hill? The car now has gravitational potential energy; as soon as it is given a small push to set it moving, it accelerates. It gains kinetic energy and at the same time it loses g.p.e.

Figure 5.12 The roller-coaster car accelerates as it comes downhill. It's even more exciting if it runs through water.

As the car runs along the roller-coaster track (Figure 5.13), its energy changes.

1 At the top of the first hill, it has the most g.p.e.
2 As it runs downhill, its g.p.e. decreases and its k.e. increases.
3 At the bottom of the hill, all of its g.p.e. has been changed to k.e. and heat and sound energy.
4 As it runs back uphill, the force of gravity slows it down. k.e. is being changed to g.p.e.

Inevitably, some energy is lost by the car. There is friction with the track, and air resistance. So the car cannot return to its original height. That is why the second hill must be slightly lower than the first. It is fun if the car runs through a trough of water, but that takes even more energy, and the car cannot rise so high. There are many situations where an object's energy changes between gravitational potential energy and kinetic energy. For example:

- a high diver falling towards the water – g.p.e. changes to k.e.
- a ball is thrown upwards – k.e. changes to g.p.e.
- a child on a swing – energy changes back and forth between g.p.e. and k.e.

Down, up, down – energy changes

When an object falls, it speeds up. Its g.p.e. decreases and its k.e. increases. Energy is being transformed from gravitational potential energy to kinetic energy. Some energy is likely to be lost, usually as heat because of air resistance. However, if no energy is lost in the process, we have:

decrease in g.p.e. = gain in k.e.

We can use this idea to solve a variety of problems, as illustrated by Worked example 4.

WORKED EXAMPLE

4 A pendulum consists of a brass sphere of mass 5.0 kg hanging from a long string (see Figure 5.14). The sphere is pulled to the side so that it is 0.15 m above its lowest position. It is then released. How fast will it be moving when it passes through the lowest point along its path?

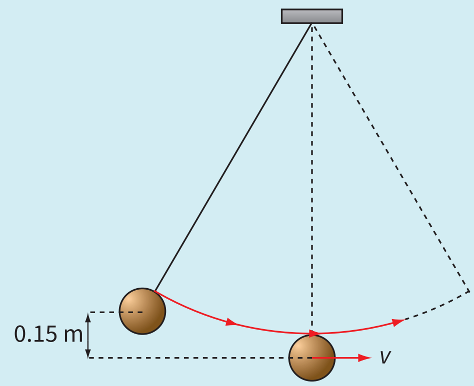


Figure 5.14 For Worked example 4.

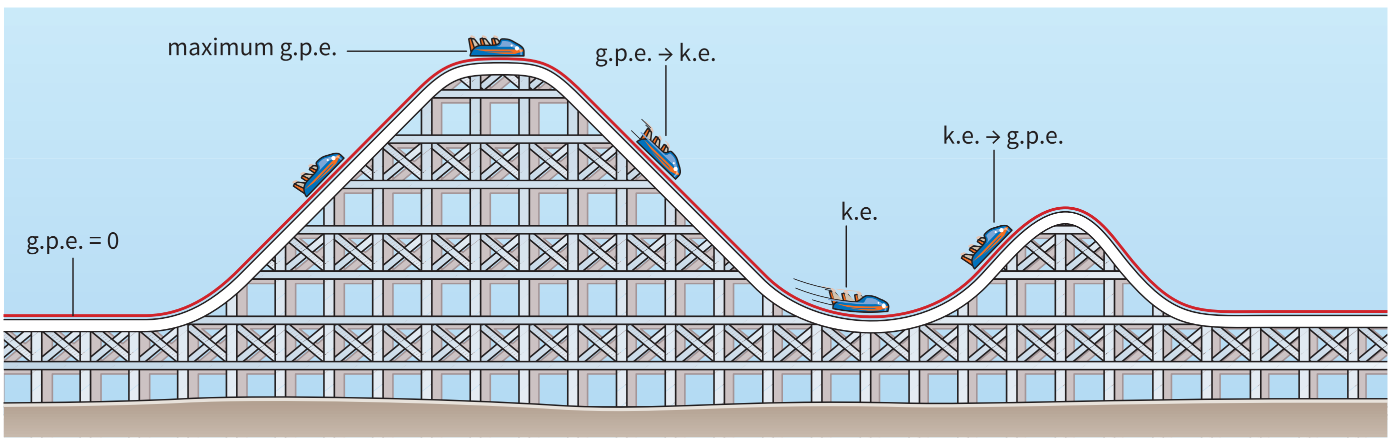


Figure 5.13 Energy changes along a roller-coaster.

WORKED EXAMPLE (continued)

Step 1 Calculate the loss in g.p.e. as the sphere falls from its highest position:

$E_p = mgh = 5.0 \times 9.81 \times 0.15 = 7.36\,J$

Step 2 The gain in the sphere's k.e. is 7.36 J. We can use this to calculate the sphere's speed. First calculate v^2, then v:

$$\tfrac{1}{2}mv^2 = 7.36$$

$$\tfrac{1}{2} \times 5.0 \times v^2 = 7.36$$

$$v^2 = 2 \times \frac{7.36}{5.0} = 2.944$$

$$v = \sqrt{2.944} \approx 1.72\,m\,s^{-1} \approx 1.7\,m\,s^{-1}$$

Note that we would obtain the same result in Worked example 4 no matter what the mass of the sphere. This is because both k.e. and g.p.e. depend on mass m. If we write:

change in g.p.e. = change in k.e.

$$mgh = \frac{1}{2}mv^2$$

we can cancel m from both sides. Hence:

$$gh = \frac{v^2}{2}$$

$$v^2 = 2gh$$

Therefore:

$$v = \sqrt{2gh}$$

The final speed v only depends on g and h. The mass m of the object is irrelevant. This is not surprising; we could use the same equation to calculate the speed of an object falling from height h. An object of small mass gains the same speed as an object of large mass, provided air resistance has no effect.

Energy transfers

Climbing bars

If you are going to climb a mountain, you will need a supply of energy. This is because your gravitational potential energy is greater at the top of the mountain than at the base. A good supply of energy would be some bars of chocolate. Each bar supplies 1200 kJ. Suppose your weight is 600 N and you climb a 2000 m high mountain. The work done by your muscles is:

$$\text{work done} = Fs = 600 \times 2000 = 1200\,kJ$$

So one bar of chocolate will do the trick. Of course, in reality, it would not. Your body is inefficient. It cannot convert 100% of the energy from food into gravitational potential energy. A lot of energy is wasted as your muscles warm up, you perspire, and your body rises and falls as you walk along the path. Your body is perhaps only 5% efficient as far as climbing is concerned, and you will need to eat 20 chocolate bars to get you to the top of the mountain. And you will need to eat more to get you back down again.

Many energy transfers are inefficient. That is, only part of the energy is transferred to where it is wanted. The rest is wasted, and appears in some form that is not wanted (such as waste heat), or in the wrong place. You can determine the efficiency of any device or system using the following equation:

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\%$$

A car engine is more efficient than a human body, but not much more. Figure 5.16 shows how this can be represented by a Sankey diagram. The width of the arrow represents the fraction of the energy which is transformed to each new form. In the case of a car engine, we want it to provide

QUESTIONS

12 Re-work Worked example 4 for a brass sphere of mass 10 kg, and show that you get the same result. Repeat with any other value of mass.

13 Calculate how much gravitational potential energy is lost by an aircraft of mass 80 000 kg if it descends from an altitude of 10 000 m to an altitude of 1000 m. What happens to this energy if the pilot keeps the aircraft's speed constant?

14 A high diver (see Figure 5.15) reaches the highest point in her jump with her centre of gravity 10 m above the water. Assuming that all her gravitational potential energy becomes kinetic energy during the dive, calculate her speed just before she enters the water.

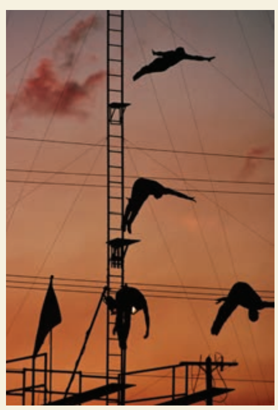


Figure 5.15 A high dive is an example of converting (transforming) gravitational potential energy to kinetic energy.

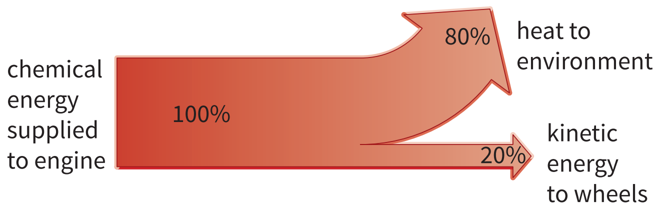


Figure 5.16 We want a car engine to supply kinetic energy. This Sankey diagram shows that only 20% of the energy supplied to the engine ends up as kinetic energy – it is 20% efficient.

kinetic energy to turn the wheels. In practice, 80% of the energy is transformed into heat: the engine gets hot, and heat escapes into the surroundings. So the car engine is only 20% efficient.

We have previously considered situations where an object is falling, and all of its gravitational potential energy changes to kinetic energy. In Worked example 5, we will look at a similar situation, but in this case the energy change is not 100% efficient.

Conservation of energy

Where does the lost energy from the water in the reservoir go? Most of it ends up warming the water, or warming the pipes that the water flows through. The outflow of water is probably noisy, so some sound is produced.

Here, we are assuming that all of the energy ends up somewhere. None of it disappears. We assume the same thing when we draw a Sankey diagram. The total thickness of the arrow remains constant. We could not have an arrow which got thinner (energy disappearing) or thicker (energy appearing out of nowhere).

We are assuming that energy is conserved. This is a principle, known as the **principle of conservation of energy**, which we expect to apply in all situations.

> Energy cannot be created or destroyed. It can only be converted from one form to another.

We should always be able to add up the total amount of energy at the beginning, and be able to account for it all at the end. We cannot be sure that this is always the case, but we expect it to hold true.

We have to think about energy changes within a closed system; that is, we have to draw an imaginary boundary around all of the interacting objects which are involved in an energy transfer.

WORKED EXAMPLE

5 Figure 5.17 shows a dam which stores water. The outlet of the dam is 20 m below the surface of the water in the reservoir. Water leaving the dam is moving at $16\,\mathrm{m\,s^{-1}}$. Calculate the percentage of the gravitational potential energy that is lost when converted into kinetic energy.

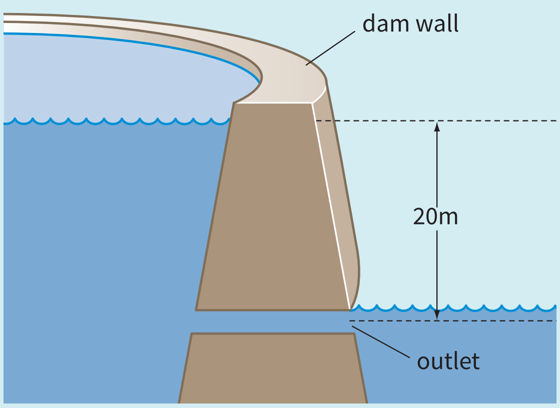


Figure 5.17 Water stored behind the dam has gravitational potential energy; the fast-flowing water leaving the foot of the dam has kinetic energy.

Step 1 We will picture 1 kg of water, starting at the surface of the lake (where it has g.p.e., but no k.e.) and flowing downwards and out at the foot (where it has k.e., but less g.p.e.). Then:

change in g.p.e. of water between surface and outflow

$$= mgh = 1 \times 9.81 \times 20 = 196\,\mathrm{J}$$

Step 2 Calculate the k.e. of 1 kg of water as it leaves the dam:

$$\text{k.e. of water leaving dam} = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 1 \times (16)^2 = 128\,\mathrm{J}$$

Step 3 For each kilogram of water flowing out of the dam, the loss of energy is:

$$\text{loss} = 196 - 128 = 68\,\mathrm{J}$$

$$\text{percentage loss} = \frac{68}{196} \times 100\% \approx 35\%$$

If you wanted to use this moving water to generate electricity, you would have already lost more than a third of the energy which it stores when it is behind the dam.

Sometimes, applying the principle of conservation of energy can seem like a scientific fiddle. When physicists were investigating radioactive decay involving beta particles, they found that the particles after the decay had less energy in total than the particles before. They guessed that there was another, invisible particle which was carrying away the missing energy. This particle, named the neutrino, was proposed by the theoretical physicist Wolfgang Pauli in 1931. The neutrino was not detected by experimenters until 25 years later.

Although we cannot prove that energy is always conserved, this example shows that the principle of conservation of energy can be a powerful tool in helping us to understand what is going on in nature, and that it can help us to make fruitful predictions about future experiments.

QUESTION

15 A stone falls from the top of a cliff, 80 m high. When it reaches the foot of the cliff, its speed is 38 m s^{-1}.

a Calculate the proportion of the stone's initial g.p.e. that is converted to k.e.

b What happens to the rest of the stone's initial energy?

Power

The word **power** has several different meanings – political power, powers of ten, electrical power from power stations. In physics, it has a specific meaning which is related to these other meanings. Figure 5.18 illustrates what we mean by power in physics.

The lift shown in Figure 5.18 can lift a heavy load of people. The motor at the top of the building provides a force to raise the lift car, and this force does work against the force of gravity. The motor transfers energy to the lift car. The **power** P of the motor is the rate at which it does work. Power is defined as the rate of work done. As a word equation, power is given by:

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

or

$$P = \frac{W}{t}$$

where W is the work done in a time t.

Units of power: the watt

Power is measured in watts, named after James Watt, the Scottish engineer famous for his development of the steam

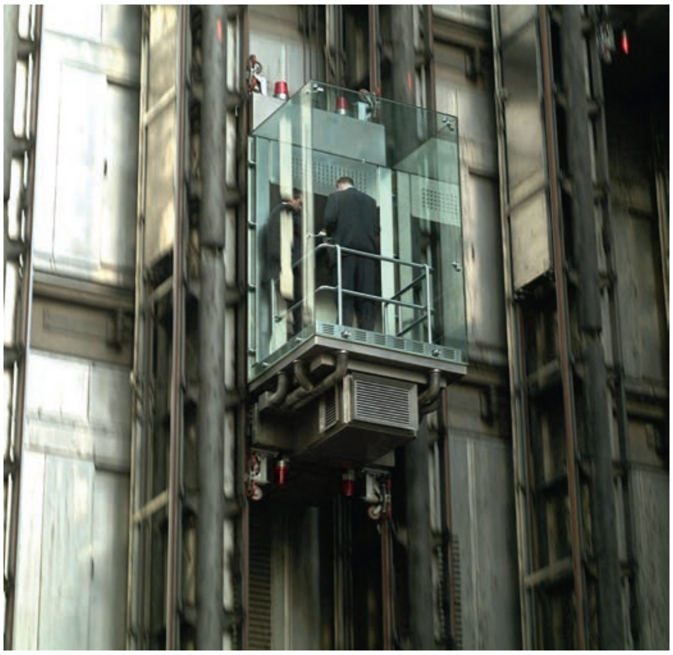


Figure 5.18 A lift needs a powerful motor to raise the car when it has a full load of people. The motor does many thousands of joules of work each second.

engine in the second half of the 18th century. The watt is defined as a rate of working of 1 joule per second. Hence:

1 watt = 1 joule per second

or

$1\,\text{W} = 1\,\text{J}\,\text{s}^{-1}$

In practice we also use kilowatts (kW) and megawatts (MW).

1000 watts = 1 kilowatt (1 kW)

1 000 000 watts = 1 megawatt (1 MW)

You are probably familiar with the labels on light bulbs which indicate their power in watts, for example 60 W or 10 W. The values of power on the labels tell you about the energy transferred by an electrical current, rather than by a force doing work.

QUESTIONS

16 Calculate how much work is done by a 50 kW car engine in a time of 1.0 minute.

17 A car engine does 4200 kJ of work in one minute. Calculate its output power, in kilowatts.

18 A particular car engine provides a force of 700 N when the car is moving at its top speed of 40 m s^{-1}.

a Calculate how much work is done by the car's engine in one second.

b State the output power of the engine.

WORKED EXAMPLE

6 The motor of the lift shown in Figure 5.18 provides a force of 20 kN; this force is enough to raise the lift by 18 m in 10 s. Calculate the output power of the motor.

Step 1 First, we must calculate the work done:

work done = force × distance moved

= 20 × 18 = 360 kJ

Step 2 Now we can calculate the motor's output power:

$$\text{power} = \frac{\text{work done}}{\text{time taken}} = \frac{360 \times 10^3}{10} = 36\,\text{kW}$$

Hint: *Take care not to confuse the two uses of the letter 'W':*

W = *watt (a unit)*

W = work done (a quantity)

So the lift motor's power is 36 kW. Note that this is its mechanical power output. The motor cannot be 100% efficient since some energy is bound to be wasted as heat due to friction, so the electrical power input must be more than 36 kW.

Moving power

An aircraft is kept moving forwards by the force of its engines pushing air backwards. The greater the force and the faster the aircraft is moving, the greater the power supplied by its engines.

Suppose that an aircraft is moving with velocity v. Its engines provide the force F needed to overcome the drag of the air. In time t, the aircraft moves a distance s equal to $v \times t$. So the work done by the engines is:

work done = force × distance

$$W = F \times v \times t$$

and the power P $\left(= \frac{\text{work done}}{\text{time taken}}\right)$ is given by:

$$P = \frac{W}{t} = \frac{F \times v \times t}{t}$$

and we have:

$$P = F \times v$$

power = force × velocity

It may help to think of this equation in terms of units. The right-hand side is in N × m s^{-1}, and N m is the same as J. So the right-hand side has units of J s^{-1}, or W, the unit of power. If you look back to Question 18 above, you will see that, to find the power of the car engine, rather than considering the work done in 1 s, we could simply have multiplied the engine's force by the car's speed.

Human power

Our energy supply comes from our food. A typical diet supplies 2000–3000 kcal (kilocalories) per day. This is equivalent (in SI units) to about 10 MJ of energy. We need this energy for our daily requirements – keeping warm, moving about, brainwork and so on. We can determine the average power of all the activities of our body:

average power = 10 MJ per day

$$= 10 \times \frac{10^6}{86\,400} = 116\,\text{W}$$

So we dissipate energy at the rate of about 100 W. We supply roughly as much energy to our surroundings as a 100 W light bulb. Twenty people will keep a room as warm as a 2 kW electric heater.

Note that this is our average power. If you are doing some demanding physical task, your power will be greater. This is illustrated in Worked example 7.

Note also that the human body is not a perfectly efficient system; a lot of energy is wasted when, for example, we lift a heavy load. We might increase an object's g.p.e. by 1000 J when we lift it, but this might require five or ten times this amount of energy to be expended by our bodies.

QUESTION

19 In an experiment to measure a student's power, she times herself running up a flight of steps. Use the data below to work out her useful power.

number of steps = 28

height of each step = 20 cm

acceleration of free fall = 9.81 m s^{-2}

mass of student = 55 kg

time taken = 5.4 s

WORKED EXAMPLE

7 A person who weighs 500 N runs up a flight of stairs in 5.0 s (Figure 5.19). Their gain in height is 3.0 m. Calculate the rate at which work is done against the force of gravity.

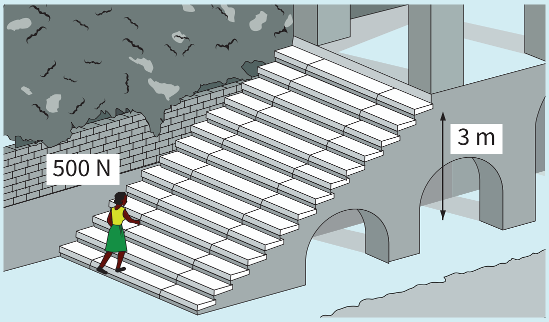


Figure 5.19 Running up stairs can require a high rate of doing work. You may have investigated your own power in this way.

Step 1 Calculate the work done against gravity:

work done $W = F \times s = 500 \times 3.0 = 1500$ J

Step 2 Now calculate the power:

power $P = \frac{W}{t} = \frac{1500}{5.0} = 300$ W

So, while the person is running up the stairs, they are doing work against gravity at a greater rate than their average power – perhaps three times as great. And, since our muscles are not very efficient, they need to be supplied with energy even faster, perhaps at a rate of 1 kW. This is why we cannot run up stairs all day long without greatly increasing the amount we eat. The inefficiency of our muscles also explains why we get hot when we exert ourselves.

Summary

- The work done W when a force F moves through a displacement s in the direction of the force:

  $W = Fs$ or $W = Fs\cos\theta$

  where θ is the angle between the force and the displacement.
- A joule is defined as the work done (or energy transferred) when a force of 1 N moves a distance of 1 m in the direction of the force.
- The work done W by a gas at pressure p when it expands:

  $W = p\Delta V$

  where ΔV is the increase in its volume.
- When an object of mass m rises through a height h, its gravitational potential energy E_p increases by an amount:

  $E_p = mgh$
- The kinetic energy E_k of a body of mass m moving at speed v is:

  $E_k = \frac{1}{2}mv^2$
- The principle of conservation of energy states that, for a closed system, energy can be transformed to other forms but the total amount of energy remains constant.
- The efficiency of a device or system is determined using the equation:

  $$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\%$$
- Power is the rate at which work is done (or energy is transferred):

  $P = \frac{W}{t}$ and $P = Fv$
- A watt is defined as a rate of transfer of energy of one joule per second.

End-of-chapter questions

1 In each case below, discuss the energy changes taking place:

a An apple falling towards the ground [1]

b A car decelerating when the brakes are applied [1]

c A space probe falling towards the surface of a planet. [1]

2 A 120 kg crate is dragged along the horizontal ground by a 200 N force acting at an angle of 30° to the horizontal, as shown in Figure 5.20. The crate moves along the surface with a constant velocity of 0.5 m s^{-1}. The 200 N force is applied for a time of 16 s.

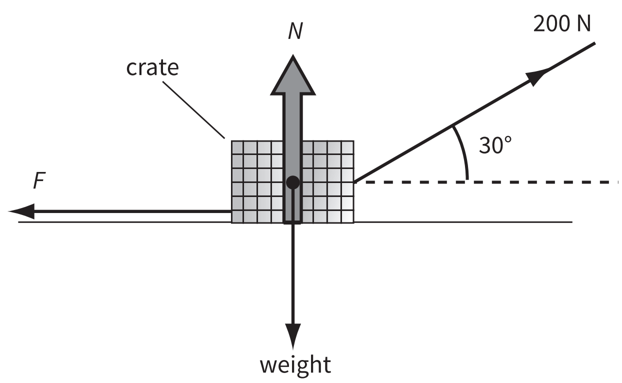


Figure 5.20 For End-of-chapter Question 2.

a Calculate the work done on the crate by:

i the 200 N force [3]

ii the weight of the crate [2]

iii the normal contact force *N*. [2]

b Calculate the rate of work done against the frictional force *F*. [2]

3 Which of the following has greater kinetic energy?

- A 20-tonne truck travelling at a speed of 30 m s^{-1}
- A 1.2 g dust particle travelling at 150 km s^{-1} through space. [3]

4 A 950 kg sack of cement is lifted to the top of a building 50 m high by an electric motor.

a Calculate the increase in the gravitational potential energy of the sack of cement. [2]

b The output power of the motor is 4.0 kW. Calculate how long it took to raise the sack to the top of the building. [2]

c The electrical power transferred by the motor is 6.9 kW. In raising the sack to the top of the building, how much energy is wasted in the motor as heat? [3]

5 a Define **power** and state its unit. [2]

b Write a word equation for the kinetic energy of a moving object. [1]

c A car of mass 1100 kg starting from rest reaches a speed of 18 m s^{-1} in 25 s. Calculate the average power developed by the engine of the car. [2]

6 A cyclist pedals a long slope which is at 5.0° to the horizontal (Figure 5.21). The cyclist starts from rest at the top of the slope and reaches a speed of 12 m s^{-1} after a time of 67 s, having travelled 40 m down the slope. The total mass of the cyclist and bicycle is 90 kg.

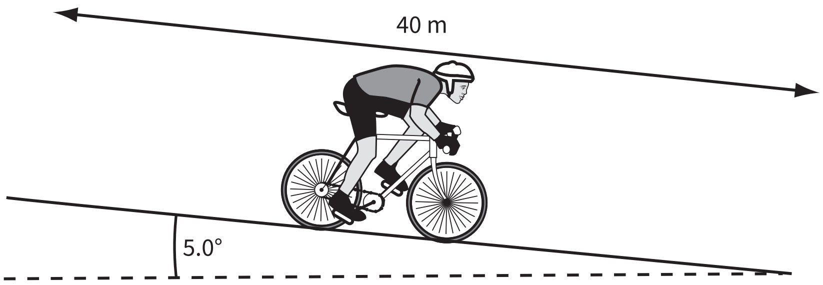


Figure 5.21 For End-of-chapter Question 6.

a Calculate:

i the loss in gravitational potential energy as he travels down the slope [3]

ii the increase in kinetic energy as he travels down the slope. [2]

b **i** Use your answers to **a** to determine the useful power output of the cyclist. [3]

ii Suggest **one** reason why the actual power output of the cyclist is larger than your value in **i**. [2]

7 **a** Explain what is meant by **work**. [2]

b **i** Explain how the principle of conservation of energy applies to a man sliding from rest down a vertical pole, if there is a constant force of friction acting on him. [2]

ii The man slides down the pole and reaches the ground after falling a distance h = 15 m. His potential energy at the top of the pole is 1000 J. Sketch a graph to show how his gravitational potential energy E_p varies with h. Add to your graph a line to show the variation of his kinetic energy E_k with h. [3]

8 **a** Use the equations of motion to show that the kinetic energy of an object of mass m moving with velocity v is $\frac{1}{2}mv^2$. [2]

b A car of mass 800 kg accelerates from rest to a speed of 20 m s^{-1} in a time of 6.0 s.

i Calculate the average power used to accelerate the car in the first 6.0 s. [2]

ii The power passed by the engine of the car to the wheels is constant. Explain why the acceleration of the car decreases as the car accelerates. [2]

9 **a** **i** Define **potential energy**. [1]

ii Distinguish between gravitational potential energy and elastic potential energy. [2]

b Seawater is trapped behind a dam at high tide and then released through turbines. The level of the water trapped by the dam falls 10.0 m until it is all at the same height as the sea.

i Calculate the mass of seawater covering an area of 1.4×10^6 m^2 and with a depth of 10.0 m. (Density of seawater = 1030 kg m^{-3}) [1]

ii Calculate the maximum loss of potential energy of the seawater in **i** when passed through the turbines. [2]

iii The potential energy of the seawater, calculated in **ii**, is lost over a period of 6.0 hours. Estimate the average power output of the power station over this time period, given that the efficiency of the power station is 50%. [3]

Chapter 6: Momentum

Learning outcomes

You should be able to:

- define linear momentum
- state and apply the principle of conservation of momentum to collisions in one and two dimensions
- relate force to the rate of change of momentum
- discuss energy changes in perfectly elastic and inelastic collisions

Understanding collisions

To improve the safety of cars the motion of a car during a crash must be understood and the forces on the driver minimised (Figure 6.1). In this way safer cars have been developed and many lives have been saved.

In this chapter, we will explore how the idea of momentum can allow us to predict how objects move after colliding (interacting) with each other. We will also see how Newton's laws of motion can be expressed in terms of momentum.

Figure 6.1 A high-speed photograph of a crash test. The cars collide head-on at 15 m s^{-1} with dummies as drivers.

The idea of momentum

Snooker players can perform some amazing moves on the table, without necessarily knowing Newton's laws of motion – see Figure 6.2. However, the laws of physics can help us to understand what happens when two snooker balls collide or when one bounces off the side cushion of the table.

Here are some examples of situations involving collisions:

- Two cars collide head-on.
- A fast-moving car runs into the back of a slower car in front.
- A footballer runs into an opponent.
- A hockey stick strikes a ball.
- A comet or an asteroid collides with a planet as it orbits the Sun.
- The atoms of the air collide constantly with each other, and with the walls of their surroundings.

Figure 6.2 If you play pool often enough, you will be able to predict how the balls will move on the table. Alternatively, you can use the laws of physics to predict their motion.

- Electrons that form an electric current collide with the vibrating ions that make up a metal wire.
- Two distant galaxies collide over millions of years.

From these examples, we can see that collisions are happening all around us, all the time. They happen on the microscopic scale of atoms and electrons, they happen in our everyday world, and they also happen on the cosmic scale of our Universe.

Modelling collisions

Springy collisions

Figure 6.3a shows what happens when one snooker ball collides head-on with a second, stationary ball. The result can seem surprising. The moving ball stops dead. The ball initially at rest moves off with the same velocity as that of the original ball. To achieve this, a snooker player must observe two conditions:

- The collision must be head-on. (If one ball strikes a glancing blow on the side of the other, they will both move off at different angles.)
- The moving ball must not be given any spin. (Spin is an added complication which we will ignore in our present study, although it plays a vital part in the games of pool and snooker.)

You can mimic the collision of two snooker balls in the laboratory using two identical trolleys, as shown in Figure 6.3b. The moving trolley has its spring-load released, so that the collision is springy. As one trolley runs into the other, the spring is at first compressed, and then it pushes out again to set the second trolley moving. The first trolley comes to a complete halt. The 'motion' of one trolley has been transferred to the other.

You can see another interesting result if two moving identical trolleys collide head-on. If the collision is springy, both trolleys bounce backwards. If a fast-moving trolley collides with a slower one, the fast trolley bounces back at the speed of the slow one, and the slow one bounces back at the speed of the fast one. In this collision, it is as if the velocities of the trolleys have been swapped.

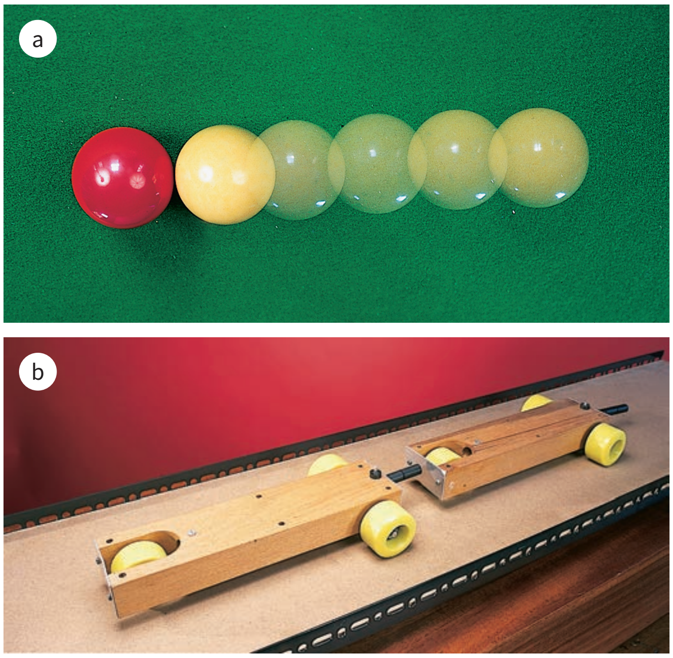


Figure 6.3 **a** The red snooker ball, coming from the left, has hit the yellow ball head-on. **b** You can do the same thing with two trolleys in the laboratory.

Sticky collisions

Figure 6.4 shows another type of collision. In this case, the trolleys have adhesive pads so that they stick together when they collide. A sticky collision like this is the opposite of a springy collision like the ones described above.

If a single moving trolley collides with an identical stationary one, they both move off together. After the collision, the speed of the combined trolleys is half that of the original trolley. It is as if the 'motion' of the original trolley has been shared between the two. If a single moving trolley collides with a stationary double trolley (twice the mass), they move off with one-third of the original velocity.

From these examples of sticky collisions, you can see that, when the mass of the trolley increases as a result of a collision, its velocity decreases. Doubling the mass halves the velocity, and so on.

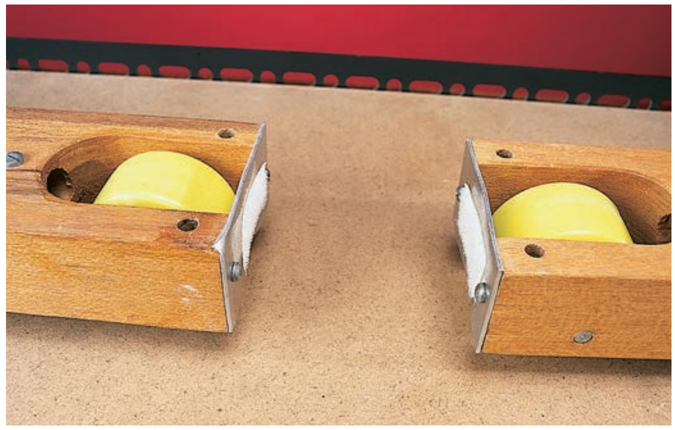


Figure 6.4 If a moving trolley sticks to a stationary trolley, they both move off together.

QUESTION

1 Here are two collisions to picture in your mind. Answer the question for each.

a Ball A, moving towards the right, collides with stationary ball B. Ball A bounces back; B moves off slowly to the right. Which has the greater mass, A or B?

b Trolley A, moving towards the right, collides with stationary trolley B. They stick together, and move off at less than half A's original speed. Which has the greater mass, A or B?

Defining linear momentum

From the examples discussed above, we can see that two quantities are important in understanding collisions:

- the mass m of the object
- the velocity v of the object.

These are combined to give a single quantity, called the **linear momentum** (or simply momentum) p of an object. The momentum of an object is defined as the product of the mass of the object and its velocity. Hence:

momentum = mass × velocity

$$p = mv$$

The unit of momentum is $\mathrm{kg\,m\,s^{-1}}$. There is no special name for this unit in the SI system.

Momentum is a vector quantity because it is a product of a vector quantity (velocity) and a scalar quantity (mass). Momentum has both magnitude and direction. Its direction is the same as the direction of the object's velocity.

In the earlier examples, we described how the 'motion' of one trolley appeared to be transferred to a second trolley, or shared with it. It is more correct to say that it is the trolley's momentum that is transferred or shared. (Strictly speaking, we should refer to linear momentum, because there is another quantity called **angular momentum** which is possessed by spinning objects.)

As with energy, we find that momentum is also conserved. We have to consider objects which form a **closed system** – that is, no external force acts on them. The principle of **conservation of momentum** states that:

> Within a closed system, the total momentum in any direction is constant.

The principle of conservation of momentum can also be expressed as follows:

> For a closed system, in any direction:
> total momentum of objects before collision
> = total momentum of objects after collision

A group of colliding objects always has as much momentum after the collision as it had before the collision. This principle is illustrated in Worked example 1.

QUESTIONS

2 Calculate the momentum of each of the following objects:

a a 0.50 kg stone travelling at a velocity of $20\,\mathrm{m\,s^{-1}}$

b a 25 000 kg bus travelling at $20\,\mathrm{m\,s^{-1}}$ on a road

c an electron travelling at $2.0 \times 10^{7}\,\mathrm{m\,s^{-1}}$.
(The mass of the electron is 9.1×10^{-31} kg.)

3 Two balls, each of mass 0.50 kg, collide as shown in Figure 6.6. Show that their total momentum before the collision is equal to their total momentum after the collision.

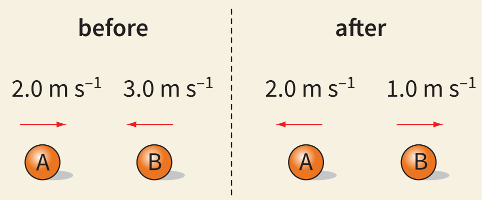


Figure 6.6 For Question 3.

WORKED EXAMPLE

1 In Figure 6.5, trolley A of mass 0.80 kg travelling at a velocity of $3.0\,\mathrm{m\,s^{-1}}$ collides head-on with a stationary trolley B. Trolley B has twice the mass of trolley A. The trolleys stick together and have a common velocity of $1.0\,\mathrm{m\,s^{-1}}$ after the collision. Show that momentum is conserved in this collision.

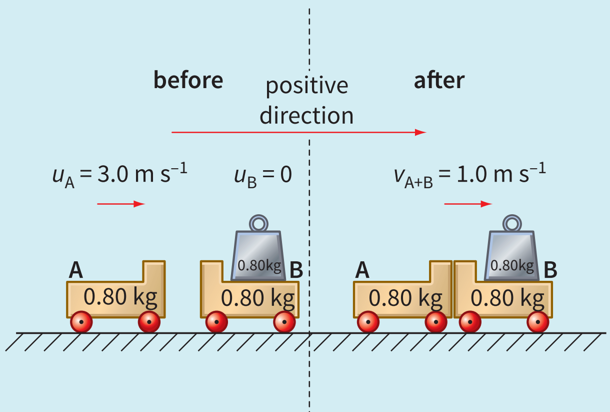


Figure 6.5 The state of trolleys A and B, before and after the collision.

Step 1 Make a sketch using the information given in the question. Notice that we need two diagrams to show the situations, one before and one after the collision. Similarly, we need two calculations – one for the momentum of the trolleys before the collision and one for their momentum after the collision.

Step 2 Calculate the momentum before the collision:

momentum of trolleys before collision

$$= m_A \times u_A + m_B \times u_B$$
$$= (0.80 \times 3.0) + 0$$
$$= 2.4\,\mathrm{kg\,m\,s^{-1}}$$

Trolley B has no momentum before the collision, because it is not moving.

Step 3 Calculate the momentum after the collision:

momentum of trolleys after collision

$$= (m_A + m_B) \times v_{A+B}$$
$$= (0.80 + 1.60) \times 1.0$$
$$= 2.4\,\mathrm{kg\,m\,s^{-1}}$$

So, both before and after the collision, the trolleys have a combined momentum of $2.4\,\mathrm{kg\,m\,s^{-1}}$. Momentum has been conserved.

Understanding collisions

The cars in Figure 6.7 have been badly damaged by a collision. The front of a car is designed to absorb the impact of the crash. It has a 'crumple zone', which collapses on impact. This absorbs most of the kinetic energy that the car had before the collision. It is better that the car's kinetic energy should be transferred to the crumple zone than to the driver and passengers.

Motor manufacturers make use of test labs to investigate how their cars respond to impacts. When a car is designed, the manufacturers combine soft, compressible materials that absorb energy with rigid structures that protect the car's occupants. Old-fashioned cars had much more rigid structures. In a collision, they were more likely to bounce back and the violent forces involved were much more likely to prove fatal.

Figure 6.7 The front of each car has crumpled in, as a result of a head-on collision.

Two types of collision

When two objects collide, they may crumple and deform. Their kinetic energy may also disappear completely as they come to a halt. This is an example of an inelastic collision. Alternatively, they may spring apart, retaining all of their kinetic energy. This is a perfectly elastic collision. In practice, in most collisions, some kinetic energy is transformed into other forms (e.g. heat or sound) and the collision is inelastic. Previously we described the collisions as being 'springy' or 'sticky'. We should now use the correct scientific terms, **perfectly elastic** and **inelastic**.

We will look at examples of these two types of collision and consider what happens to linear momentum and kinetic energy in each.

A perfectly elastic collision

Two identical objects A and B, moving at the same speed but in opposite directions, have a head-on collision, as shown in Figure 6.8. Each object bounces back with its velocity reversed. This is a perfectly elastic collision.

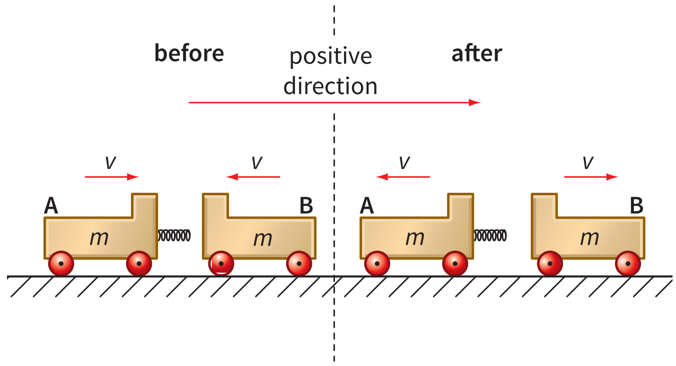


Figure 6.8 Two objects may collide in different ways: this is an elastic collision. An inelastic collision of the same two objects is shown in Figure 6.9.

You should be able to see that, in this collision, both momentum and kinetic energy are conserved. Before the collision, object A of mass m is moving to the right at speed v and object B of mass m is moving to the left at speed v. Afterwards, we still have two masses m moving with speed v, but now object A is moving to the left and object B is moving to the right. We can express this mathematically as follows:

Before the collision

object A: mass = m velocity = v momentum = mv
object B: mass = m velocity = $-v$ momentum = $-mv$

Object B has negative velocity and momentum because it is travelling in the opposite direction to object A. Therefore we have:

total momentum before collision

$$= \text{momentum of A} + \text{momentum of B}$$
$$= mv + (-mv) = 0$$

total kinetic energy before collision

$$= \text{k.e. of A} + \text{k.e. of B}$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = mv^2$$

The magnitude of the momentum of each object is the same. Momentum is a vector quantity and we have to consider the directions in which the objects travel. The combined momentum is zero. On the other hand, kinetic energy is a scalar quantity and direction of travel is irrelevant. Both objects have the same kinetic energy and therefore the combined kinetic energy is twice the kinetic energy of a single object.

After the collision

Both objects have their velocities reversed, and we have:

total momentum after collision = $(-mv) + mv = 0$

total kinetic energy after collision = $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

So the total momentum and the total kinetic energy are unchanged. They are both conserved in a perfectly elastic collision such as this.

In this collision, the objects have a **relative speed** of $2v$ before the collision. After their collision, their velocities are reversed so their relative speed is $2v$ again. This is a feature of perfectly elastic collisions.

The relative speed of approach is the speed of one object measured relative to another. If two objects are travelling directly towards each other with speed v, as measured by someone stationary on the ground, then each object 'sees' the other one approaching with a speed of $2v$. Thus if objects are travelling in opposite directions we add their speeds to find the relative speed. If the objects are travelling in the same direction then we subtract their speeds to find the relative speed.

In a perfectly elastic collision,
relative speed of approach = relative speed of separation.

An inelastic collision

In Figure 6.9, the same two objects collide, but this time they stick together after the collision and come to a halt. Clearly, the total momentum and the total kinetic energy are both zero after the collision, since neither mass is moving. We have:

	Before collision	After collision
momentum	0	0
kinetic energy	$\frac{1}{2}mv^2$	0

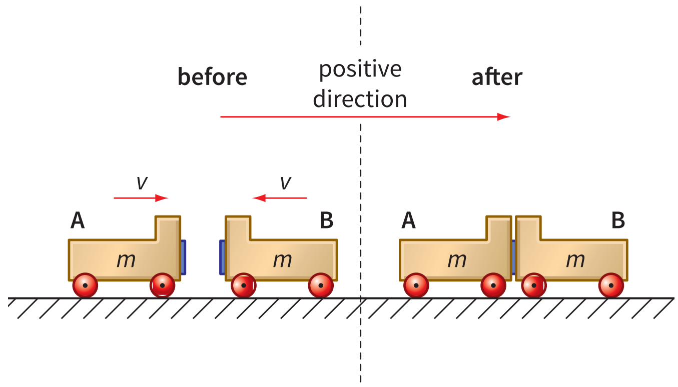


Figure 6.9 An inelastic collision between two identical objects. The trolleys are stationary after the collision.

Again we see that momentum is conserved. However, kinetic energy is not conserved. It is lost because work is done in deforming the two objects.

In fact, **momentum is always conserved** in all collisions. There is nothing else into which momentum can be converted. Kinetic energy is usually not conserved in a collision, because it can be transformed into other forms of energy – sound energy if the collision is noisy, and the energy involved in deforming the objects (which usually ends up as internal energy – they get warmer). Of course, the total amount of energy remains constant, as prescribed by the principle of conservation of energy.

QUESTION

4 Copy Table 6.1 below, choosing the correct words from each pair.

Type of collision	perfectly elastic	inelastic
Momentum	conserved / not conserved	conserved / not conserved
Kinetic energy	conserved / not conserved	conserved / not conserved
Total energy	conserved / not conserved	conserved / not conserved

Table 6.1 For Question 4.

Solving collision problems

We can use the idea of conservation of momentum to solve numerical problems, as illustrated by Worked example 2.

WORKED EXAMPLE

2 In the game of bowls, a player rolls a large ball towards a smaller, stationary ball. A large ball of mass 5.0 kg moving at 10.0 m s^{-1} strikes a stationary ball of mass 1.0 kg. The smaller ball flies off at 10.0 m s^{-1}.

a Determine the final velocity of the large ball after the impact.

b Calculate the kinetic energy 'lost' in the impact.

Step 1 Draw two diagrams, showing the situations before and after the collision. Figure 6.10 shows the values of masses and velocities; since we don't know the velocity of the large ball after the collision, this is shown as *v*. The direction from left to right has been assigned the 'positive' direction.

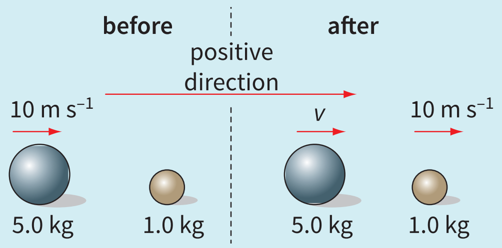


Figure 6.10 When solving problems involving collisions, it is useful to draw diagrams showing the situations before and after the collision. Include the values of all the quantities that you know.

Step 2 Using the principle of conservation of momentum, set up an equation and solve for the value of *v*:

total momentum before collision
= total momentum after collision

$$(5.0 \times 10) + (1.0 \times 0) = (5.0 \times v) + (1.0 \times 10)$$

$$50 + 0 = 5.0v + 10$$

$$v = \frac{40}{5.0} = 8.0\,\text{m s}^{-1}$$

So the speed of the large ball decreases to 8.0 m s^{-1} after the collision. Its direction of motion is unchanged – the velocity remains positive.

Step 3 Knowing the large ball's final velocity, calculate the change in kinetic energy during the collision:

total k.e. before collision $= \frac{1}{2} \times 5.0 \times 10^2 + 0 = 250\,\text{J}$

total k.e. after collision $= \frac{1}{2} \times 5.0 \times 8.0^2 + \frac{1}{2} \times 1.0 \times 10^2$
$= 210\,\text{J}$

k.e. 'lost' in the collision $= 250\,\text{J} - 210\,\text{J} = 40\,\text{J}$

This lost kinetic energy will appear as internal energy (the two balls get warmer) and as sound energy (we hear the collision between the balls).

QUESTIONS

5 Figure 6.11 shows two identical balls A and B about to make a head-on collision. After the collision, ball A rebounds at a speed of 1.5 m s^{-1} and ball B rebounds at a speed of 2.5 m s^{-1}. The mass of each ball is 4.0 kg.

a Calculate the momentum of each ball before the collision.

b Calculate the momentum of each ball after the collision.

c Is the momentum conserved in the collision?

d Show that the total kinetic energy of the two balls is conserved in the collision.

e Show that the relative speed of the balls is the same before and after the collision.

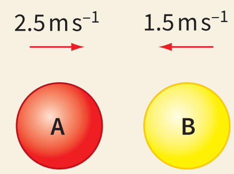


Figure 6.11 For Question 5.

6 A trolley of mass 1.0 kg is moving at 2.0 m s^{-1}. It collides with a stationary trolley of mass 2.0 kg. This second trolley moves off at 1.2 m s^{-1}.

a Draw 'before' and 'after' diagrams to show the situation.

b Use the principle of conservation of momentum to calculate the speed of the first trolley after the collision. In what direction does it move?

Explosions and crash-landings

There are situations where it may appear that momentum is being created out of nothing, or that it is disappearing without trace. Do these contradict the principle of conservation of momentum?

The rockets shown in Figure 6.12 rise high into the sky. As they start to fall, they send out showers of chemical packages, each of which explodes to produce a brilliant sphere of burning chemicals. Material flies out in all directions to create a spectacular effect.

Does an explosion create momentum out of nothing? The important point to note here is that the burning material spreads out equally in all directions. Each tiny spark has momentum, but for every spark, there is another moving in the opposite direction, i.e. with opposite momentum. Since momentum is a vector quantity, the total amount of momentum created is zero.

Figure 6.12 These exploding rockets produce a spectacular display of bright sparks in the night sky.

At the same time, kinetic energy is created in an explosion. Burning material flies outwards; its kinetic energy has come from the chemical potential energy stored in the chemical materials before they burn.

More fireworks

A roman candle fires a jet of burning material up into the sky. This is another type of explosion, but it doesn't send material in all directions. The firework tube directs the material upwards. Has momentum been created out of nothing here?

Again, the answer is no. The chemicals have momentum upwards, but at the same time, the roman candle pushes downwards on the Earth. An equal amount of downwards momentum is given to the Earth. Of course, the Earth is massive, and we don't notice the tiny change in its velocity which results.

Down to Earth

If you push a large rock over a cliff, its speed increases as it falls. Where does its momentum come from? And when it lands, where does its momentum disappear to?

The rock falls because of the pull of the Earth's gravity on it. This force is its weight and it makes the rock accelerate towards the Earth. Its weight does work and the rock gains kinetic energy. It gains momentum downwards. Something must be gaining an equal amount of momentum in the opposite (upward) direction. It is the Earth, which starts to move upwards as the rock falls downwards. The mass of the Earth is so great that its change in velocity is small – far too small to be noticeable.

When the rock hits the ground, its momentum becomes zero. At the same instant, the Earth also stops moving upwards. The rock's momentum cancels out the Earth's momentum. At all times during the rock's fall and crash-landing, momentum has been conserved.

If a rock of mass 60 kg is falling towards the Earth at a speed of $20\,\mathrm{m\,s^{-1}}$, how fast is the Earth moving towards it? Figure 6.13 shows the situation. The mass of the Earth is 6.0×10^{24} kg. We have:

total momentum of Earth and rock = 0

Hence:

$$(60 \times 20) + (6.0 \times 10^{24} \times v) = 0$$

$$v = -2.0 \times 10^{-22}\,\mathrm{m\,s^{-1}}$$

The minus sign shows that the Earth's velocity is in the opposite direction to that of the rock. The Earth moves very slowly indeed. In the time of the rock's fall, it will move much less than the diameter of the nucleus of an atom!

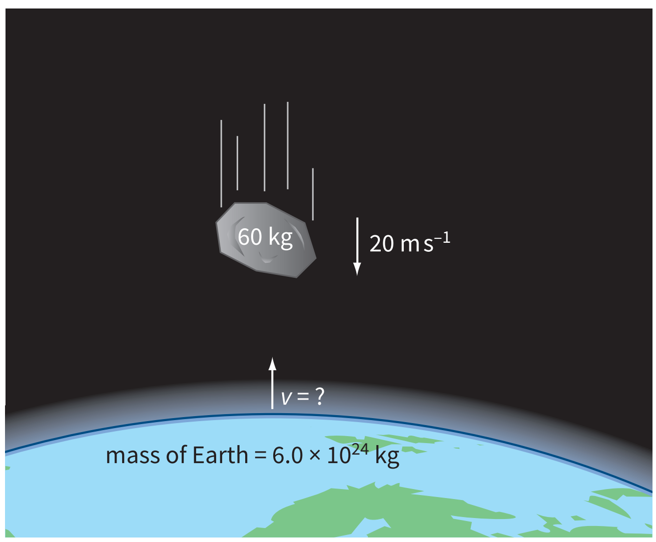


Figure 6.13 The rock and Earth gain momentum in opposite directions.

QUESTIONS

7 Discuss whether momentum is conserved in each of the following situations.

a A star explodes in all directions – a supernova.

b You jump up from a trampoline. As you go up, your speed decreases; as you come down again, your speed increases.

8 A ball of mass 0.40 kg is thrown at a wall. It strikes the wall with a speed of $1.5\,\mathrm{m\,s^{-1}}$ perpendicular to the wall and bounces off the wall with a speed of $1.2\,\mathrm{m\,s^{-1}}$. Explain the changes in momentum and energy which happen in the collision between the ball and the wall. Give numerical values where possible.

Collisions in two dimensions

It is rare that collisions happen in a straight line – in one dimension. Figure 6.14 shows a two-dimensional collision between two snooker balls. From the multiple images, we can see how the velocities of the two balls change:

- At first, the white ball is moving straight forwards. When it hits the red ball, it moves off to the right. Its speed decreases; we can see this because the images get closer together.
- The red ball moves off to the left. It moves off at a bigger angle than the white ball, but more slowly – the images are even closer together.

How can we understand what happens in this collision, using the ideas of momentum and kinetic energy?

At first, only the white ball has momentum, and this is in the forward direction. During the collision, this momentum is shared between the two balls. We can see this because each has a component of velocity in the forward direction.

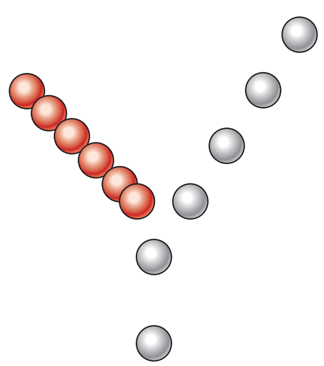


Figure 6.14 The white ball strikes the red ball a glancing blow. The two balls move off in different directions.

At the same time, each ball gains momentum in the sideways direction, because each has a sideways component of velocity – the white ball to the right, and the red ball to the left. These must be equal in magnitude and opposite in direction, otherwise we would conclude that momentum had been created out of nothing. The red ball moves at a greater angle, but its velocity is less than that of the white ball, so that the component of its velocity at right angles to the original track is the same as the white ball's.

Figure 6.15a shows the momentum of each ball before and after the collision. We can draw a vector triangle to represent the changes of momentum in this collision (Figure 6.15b). The two momentum vectors after the collision add up to equal the momentum of the white ball before the collision. The vectors form a closed triangle because momentum is conserved in this two-dimensional collision.

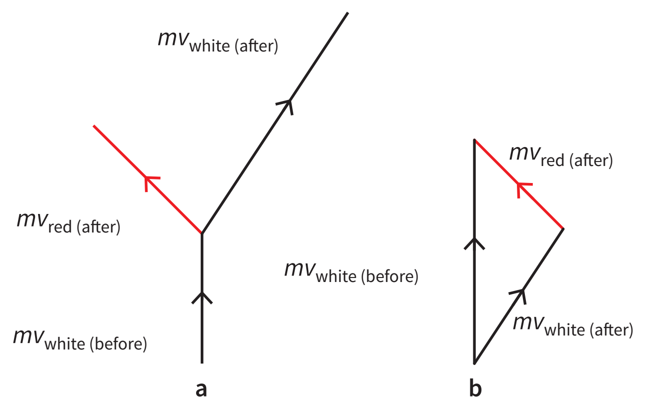


Figure 6.15 **a** These vectors represent the momenta of the colliding balls shown in Figure 6.14. **b** The closed vector triangle shows that momentum is conserved in the collision.

Components of momentum

Momentum is a vector quantity and so we can split it into components in order to solve problems.

Worked example 3 shows how to find an unknown velocity.

Worked example 4 shows how to demonstrate that momentum has been conserved in a two-dimensional collision.

WORKED EXAMPLES

3 A white ball of mass $m = 1.0$ kg and moving with initial speed $u = 0.5\,\mathrm{m\,s^{-1}}$ collides with a stationary red ball of the same mass. They move off so that each has the same speed and the angle between their paths is 90°. What is their speed?

Step 1 Draw a diagram to show the velocity vectors of the two balls, before and after the collision (Figure 6.16). We will show the white ball initially travelling along the *y*-direction.

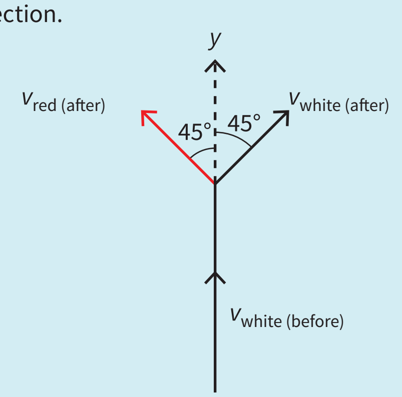


Figure 6.16 Velocity vectors for the white and red balls.

Because we know that the two balls have the same final speed *v*, their paths must be symmetrical about the *y*-direction. Since their paths are at 90° to one other, each must be at 45° to the *y*-direction.

Step 2 We know that momentum is conserved in the *y*-direction. Hence we can say:

initial momentum of white ball in *y*-direction
= final component of momentum of white ball in *y*-direction
+ final component of momentum of red ball in *y*-direction

This is easier to understand using symbols:

$mu = mv_y + mv_y$

where v_y is the component of *v* in the *y*-direction. The right-hand side of this equation has two identical terms, one for the white ball and one for the red. We can simplify the equation to give:

$mu = 2mv_y$

Step 3 The component of *v* in the *y*-direction is $v\cos 45°$. Substituting this, and including values of *m* and *u*, gives

$0.5 = 2v\cos 45°$

and hence

$$v = \frac{0.5}{2\cos 45°} \approx 0.354\,\mathrm{m\,s^{-1}}$$

So each ball moves off at $0.354\,\mathrm{m\,s^{-1}}$ at an angle of 45° to the initial direction of the white ball.

4 Figure 6.17 shows the momentum vectors for particles 1 and 2, before and after a collision. Show that momentum is conserved in this collision.

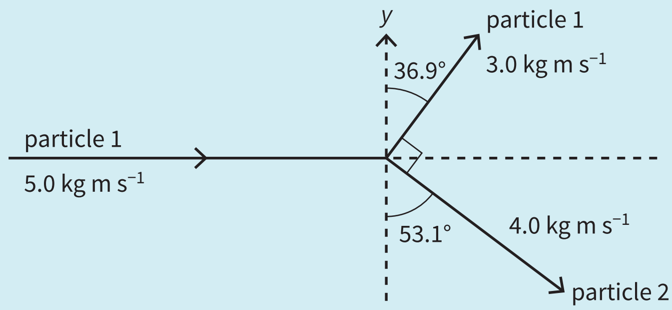


Figure 6.17 Momentum vectors: particle 1 has come from the left and collided with particle 2.

Step 1 Consider momentum changes in the *y*-direction.

Before collision:

momentum = 0

(because particle 1 is moving in the *x*-direction and particle 2 is stationary).

After collision:

component of momentum of particle 1
$= 3.0\cos 36.9° \approx 2.40\,\mathrm{kg\,m\,s^{-1}}$ upwards

component of momentum of particle 2
$= 4.0\cos 53.1° \approx 2.40\,\mathrm{kg\,m\,s^{-1}}$ downwards

These components are equal and opposite and hence their sum is zero. Hence momentum is conserved in the *y*-direction.

Step 2 Consider momentum changes in the *x*-direction.

Before collision: momentum $= 5.0\,\mathrm{kg\,m\,s^{-1}}$ to the right

After collision:

component of momentum of particle 1
$= 3.0\cos 53.1° \approx 1.80\,\mathrm{kg\,m\,s^{-1}}$ to the right

component of momentum of particle 2
$= 4.0\cos 36.9° \approx 3.20\,\mathrm{kg\,m\,s^{-1}}$ to the right

total momentum to the right $= 5.0\,\mathrm{kg\,m\,s^{-1}}$

Hence momentum is conserved in the *x*-direction.

Step 3 An alternative approach would be to draw a vector triangle similar to Figure 6.15b. In this case, the numbers have been chosen to make this easy; the vectors form a 3–4–5 right-angled triangle.

Because the vectors form a closed triangle, we can conclude that:

momentum before collision = momentum after collision i.e. momentum is conserved.

QUESTIONS

9 A snooker ball strikes a stationary ball. The second ball moves off sideways, at 60° to the initial path of the first ball.

Use the idea of conservation of momentum to explain why the first ball cannot travel in its initial direction after the collision. Illustrate your answer with a diagram.

10 Look back to Worked example 4 above. Draw the vector triangle which shows that momentum is conserved in the collision described in the question. Show the value of each angle in the triangle.

11 Figure 6.18 shows the momentum vectors for two particles, 1 and 2, before and after a collision. Show that momentum is conserved in this collision.

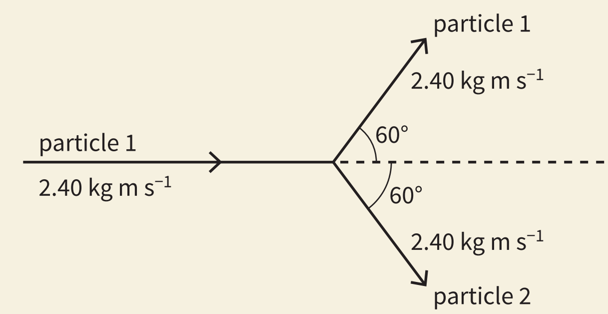


Figure 6.18 For Question 11.

12 A snooker ball collides with a second identical ball as shown in Figure 6.19.

a Determine the components of the velocity of the first ball in the x- and y-directions.

b Hence determine the components of the velocity of the second ball in the x- and y-directions.

c Hence determine the velocity (magnitude and direction) of the second ball.

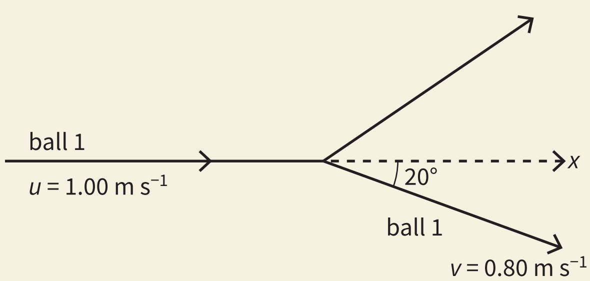


Figure 6.19 For Question 12.

Momentum and Newton's laws

The big ideas of physics are often very simple; that is to say, it takes only a few words to express them and they can be applied in many situations. However, 'simple' does not usually mean 'easy'. Concepts such as force, energy and voltage, for example, are not immediately obvious. They usually took someone to make a giant leap of imagination to first establish them. Then the community of physicists spent decades worrying away at them, refining them until they are the fundamental ideas which we use today.

Take Isaac Newton's work on motion. He published his ideas in a book commonly known as the *Principia* (see Figure 6.20); its full title translated from Latin is *Mathematical Principles of Natural Philosophy*.

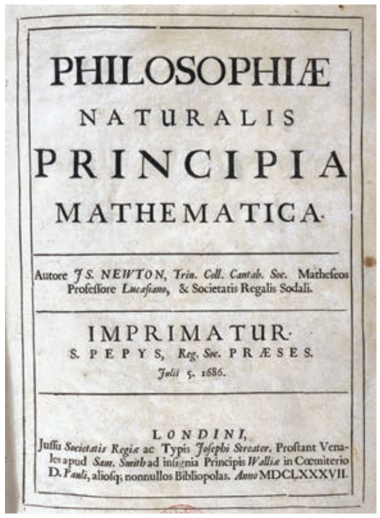


Figure 6.20 The title page of Newton's *Principia*, in which he outlined his theories of the laws that govern the motion of objects.

The *Principia* represents the results of 20 years of thinking. Newton was able to build on Galileo's ideas and he was in correspondence with many other scientists and mathematicians. Indeed, there was an ongoing feud with Robert Hooke as to who was the first to come up with certain ideas. Among scientists, this is known as 'priority', and publication is usually taken as proof of priority.

Newton wanted to develop an understanding of the idea of 'force'. You may have been told in your early studies of science that 'a force is a push or a pull'. That doesn't tell us very much. Newton's idea was that forces are interactions between bodies and that they change the motion of the body that they act on. Forces acting on an object can produce acceleration. For an object of constant mass, this acceleration is directly proportional to the resultant force acting on the object. That is much more like a scientific definition of force.

Understanding motion

In Chapter 3, we looked at Newton's laws of motion. We can get further insight into these laws by thinking about them in terms of momentum.

Newton's first law of motion

In everyday speech, we sometimes say that something has momentum when we mean that it has a tendency to keep on moving of its own free will. An oil tanker is difficult to stop at sea, because of its momentum. We use the same word in a figurative sense: 'The election campaign is gaining momentum.' This idea of keeping on moving is just what we discussed in connection with Newton's first law of motion:

> An object will remain at rest or keep travelling at constant velocity unless it is acted on by a resultant force.

An object travelling at constant velocity has constant momentum. Hence the first law is really saying that the momentum of an object remains the same unless the object experiences an external force.

Newton's second law of motion

Newton's second law of motion links the idea of the resultant force acting on an object and its momentum. A statement of Newton's second law is:

> The resultant force acting on an object is directly proportional to the rate of change of the linear momentum of that object. The resultant force and the change in momentum are in the same direction.

Hence:

resultant force ∝ rate of change of momentum

This can be written as:

$$F \propto \frac{\Delta p}{\Delta t}$$

where F is the resultant force and Δp is the change in momentum taking place in a time interval of Δt. (Remember that the Greek letter delta, Δ, is a shorthand for 'change in …', so Δp means 'change in momentum'.) The changes in momentum and force are both vector quantities, hence these two quantities must be in the same direction.

The unit of force (the newton, N) is defined to make the constant of proportionality equal to one, so we can write the second law of motion mathematically as:

$$F = \frac{\Delta p}{\Delta t}$$

If the forces acting on an object are balanced, there is no resultant force and the object's momentum will remain constant. If a resultant force acts on an object, its momentum (velocity and/or direction) will change. The equation above gives us another way of stating Newton's second law of motion:

> The resultant force acting on an object is equal to the rate of change of its momentum. The resultant force and the change in momentum are in the same direction.

This statement effectively defines what we mean by a force; it is an interaction that causes an object's momentum to change. So, if an object's momentum is changing, there must be a force acting on it. We can find the size and direction of the force by measuring the rate of change of the object's momentum:

> force = rate of change of momentum
>
> $$F = \frac{\Delta p}{\Delta t}$$

Worked example 5 shows how to use this equation.

WORKED EXAMPLE

5 Calculate the average force acting on a 900 kg car when its velocity changes from $5.0\ \text{m s}^{-1}$ to $30\ \text{m s}^{-1}$ in a time of 12 s.

Step 1 Write down the quantities given:

$m = 900\ \text{kg}$

initial velocity $u = 5.0\ \text{m s}^{-1}$

$\Delta t = 12\ \text{s}$

Step 2 Calculate the initial momentum and the final momentum of the car:

momentum = mass × velocity

initial momentum = $mu = 900 \times 5.0 = 4500\ \text{kg m s}^{-1}$

final momentum = $mv = 900 \times 30 = 27\,000\ \text{kg m s}^{-1}$

Step 3 Use Newton's second law of motion to calculate the average force on the car:

$$F = \frac{\Delta p}{\Delta t}$$

$$= \frac{27\,500 - 4500}{12}$$

$$= 1875\ \text{N} \approx 1900\ \text{N}$$

The average force acting on the car is about 1.9 kN.

A special case of Newton's second law of motion

Imagine an object of constant mass m acted upon by a resultant force F. The force will change the momentum of the object. According to Newton's second law of motion, we have:

$$F = \frac{\Delta p}{\Delta t} = \frac{mv - mu}{t}$$

where u is the initial velocity of the object, v is the final velocity of the object and t is the time taken for the change in velocity. The mass m of the object is a constant; hence the above equation can be rewritten as:

$$F = \frac{m(v-u)}{t} = m\left(\frac{v-u}{t}\right)$$

The term in brackets on the right-hand side is the acceleration a of the object. Therefore a special case of Newton's second law is:

$$F = ma$$

We have already met this equation in Chapter 3. In Worked example 5, you could have determined the average force acting on the car using this simplified equation for Newton's second law of motion. Remember that the equation $F = ma$ is a special case of $F = \frac{\Delta p}{\Delta t}$ which only applies when the mass of the object is constant. There are situations where the mass of an object changes as it moves, for example a rocket, which burns a phenomenal amount of chemical fuel as it accelerates upwards.

Newton's third law of motion

Newton's third law of motion is about interacting objects. These could be two magnets attracting or repelling each other, two electrons repelling each other, etc. Newton's third law states:

When two bodies interact, the forces they exert on each other are equal and opposite.

How can we relate this to the idea of momentum? Picture holding two magnets, one in each hand. You gradually bring them towards each other (Figure 6.21) so that they start to attract each other. Each feels a force pulling it towards the other. The two forces are the same size, even if one magnet is stronger than the other. Indeed, one magnet could be replaced by an unmagnetised piece of steel and they would still attract each other equally.

If you release the magnets, they will gain momentum as they are pulled towards each other. One gains momentum to the left while the other gains equal momentum to the right.

Each is acted on by the same force, and for the same time. Hence momentum is conserved.

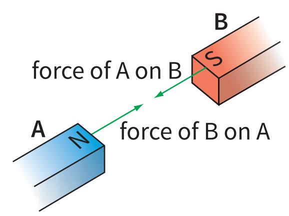


Figure 6.21 Newton's third law states that the forces these two magnets exert one each other must be equal and opposite.

QUESTIONS

13 A car of mass 1000 kg is travelling at a velocity of 10 m s^{-1}. It accelerates for 15 s, reaching a velocity of 24 m s^{-1}. Calculate:

- **a** the change in the momentum of the car in the 15 s period
- **b** the average force acting on the car as it accelerates.

14 A ball is kicked by a footballer. The average force on the ball is 240 N and the impact lasts for a time interval of 0.25 s.

- **a** Calculate the change in the ball's momentum.
- **b** State the direction of the change in momentum.

15 Water pouring from a broken pipe lands on a flat roof. The water is moving at 5.0 m s^{-1} when it strikes the roof. The water hits the roof at a rate of 10 kg s^{-1}. Calculate the force of the water hitting the roof. (Assume that the water does not bounce as it hits the roof. If it did bounce, would your answer be greater or smaller?)

16 A golf ball has a mass of 0.046 kg. The final velocity of the ball after being struck by a golf club is 50 m s^{-1}. The golf club is in contact with the ball for a time of 1.3 ms. Calculate the average force exerted by the golf club on the ball.

Summary

- Linear momentum is the product of mass and velocity:

  momentum = mass × velocity or $p = mv$

- The principle of conservation of momentum:

  For a closed system, in any direction the total momentum before an interaction (e.g. collision) is equal to the total momentum after the interaction.

- In all interactions or collisions, momentum and total energy are conserved.
- Kinetic energy is conserved in a perfectly elastic collision; relative speed is unchanged in a perfectly elastic collision.
- In an inelastic collision, kinetic energy is not conserved. It is transferred into other forms of energy (e.g. heat or sound). Most collisions are inelastic.
- Newton's first law of motion: An object will remain at rest or keep travelling at constant velocity unless it is acted on by a resultant force.
- Newton's second law of motion: The resultant force acting on a body is equal to the rate of change of its momentum:

  resultant force = rate of change of momentum

  or

  $F = \frac{\Delta p}{\Delta t}$

- Newton's third law of motion: When two bodies interact, the forces they exert on each other are equal and opposite.
- The equation $F = ma$ is a special case of Newton's second law of motion when mass m remains constant.

End-of-chapter questions

1 An object is dropped and its momentum increases as it falls toward the ground. Explain how the law of conservation of momentum and Newton's third law of motion can be applied to this situation. [2]

2 A ball of mass 2 kg, moving at $3.0\,\text{m}\,\text{s}^{-1}$, strikes a wall and rebounds with the same speed. State and explain whether there is a change in:

  a the momentum of the ball [3]

  b the kinetic energy of the ball. [1]

3 a Define **linear momentum**. [1]

  b Determine the base units of linear momentum in the SI system. [1]

  c A car of mass 900 kg starting from rest has a constant acceleration of $3.5\,\text{m}\,\text{s}^{-2}$. Calculate its momentum after it has travelled a distance of 40 m. [2]

  d Figure 6.22 shows two identical objects about to make a head-on collision. The objects stick together during the collision. Determine the final speed of the objects. State the direction in which they move. [3]

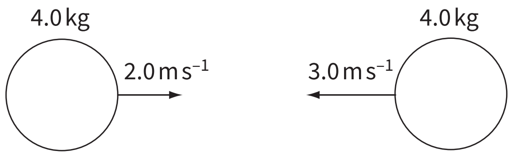


Figure 6.22 For End-of-chapter Question 3.

4 **a** Explain what is meant by an:

i elastic collision [1]

ii inelastic collision. [1]

b A snooker ball of mass 0.35 kg hits the side of a snooker table at right angles and bounces off also at right angles. Its speed before collision is 2.8 m s^{-1} and its speed after is 2.5 m s^{-1}. Calculate the change in the momentum of the ball. [2]

c Explain whether or not momentum is conserved in the situation described in **b**. [3]

5 A car of mass 1100 kg is travelling at 24 m s^{-1}. The driver applies the brakes and the car decelerates uniformly and comes to rest in 20 s.

a Calculate the change in momentum of the car. [2]

b Calculate the braking force on the car. [2]

c Determine the braking distance of the car. [2]

6 A marble of mass 100 g is moving at a speed of 0.40 m s^{-1} in the *x*-direction.

a Calculate the marble's momentum. [2]

The marble strikes a second, identical marble. Each moves off at an angle of 45° to the *x*-direction.

b Use the principle of conservation of momentum to determine the speed of each marble after the collision. [3]

c Show that kinetic energy is conserved in this collision. [2]

7 A cricket bat strikes a ball of mass 0.16 kg travelling towards it. The ball initially hits the bat at a speed of 25 m s^{-1} and returns along the same path with the same speed. The time of impact is 0.0030 s.

a Determine the change in momentum of the cricket ball. [2]

b Determine the force exerted by the bat on the ball. [2]

c Describe how the laws of conservation of energy and momentum apply to this impact and state whether the impact is elastic or inelastic. [4]

8 **a** State the principle of conservation of momentum and state the conditions under which it is valid. [2]

b An arrow of mass 0.25 kg is fired horizontally towards an apple of mass 0.10 kg which is hanging on a string (Figure 6.23).

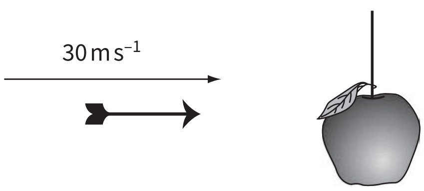


Figure 6.23 For End-of-chapter Question 8.

The horizontal velocity of the arrow as it enters the apple is 30 m s^{-1}. The apple was initially at rest and the arrow sticks in the apple.

i Calculate the horizontal velocity of the apple and arrow immediately after the impact. [2]

ii Calculate the change in momentum of the arrow during the impact. [2]

iii Calculate the change in total kinetic energy of the arrow and apple during the impact. [2]

iv An identical arrow is fired at the centre of a stationary ball of mass 0.25 kg. The collision is perfectly elastic. Describe what happens and state the relative speed of separation of the arrow and the ball. [2]

9 **a** State what is meant by:

i a perfectly elastic collision [1]

ii a completely inelastic collision. [1]

b A stationary uranium nucleus disintegrates, emitting an alpha-particle of mass 6.65×10^{-27} kg and another nucleus X of mass 3.89×10^{-25} kg (Figure 6.24).

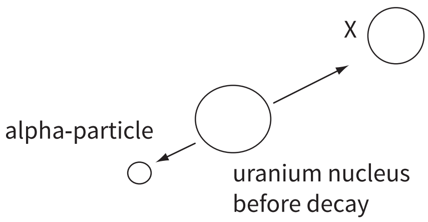


Figure 6.24 For End-of-chapter Question 9.

i Explain why the alpha-particle and nucleus X must be emitted in exactly opposite directions. [2]

ii Using the symbols v_α and v_X for velocities, write an equation for the conservation of momentum in this disintegration. [1]

iii Using your answer to **ii**, calculate the ratio v_α / v_X after the disintegration. [1]

10 **a** State **two** quantities that are conserved in an elastic collision. [1]

b A machine gun fires bullets of mass 0.014 kg at a speed of $640\,\text{m}\,\text{s}^{-1}$.

i Calculate the momentum of each bullet as it leaves the gun. [1]

ii Explain why a soldier holding the machine gun experiences a force when the gun is firing. [2]

iii The maximum steady horizontal force that a soldier can exert on the gun is 140 N. Calculate the maximum number of bullets that the gun can fire in one second. [2]

11 Two railway trucks are travelling in the same direction and collide. The mass of truck X is 2.0×10^4 kg and the mass of truck Y is 3.0×10^4 kg. Figure 6.25 shows how the velocity of each truck varies with time.

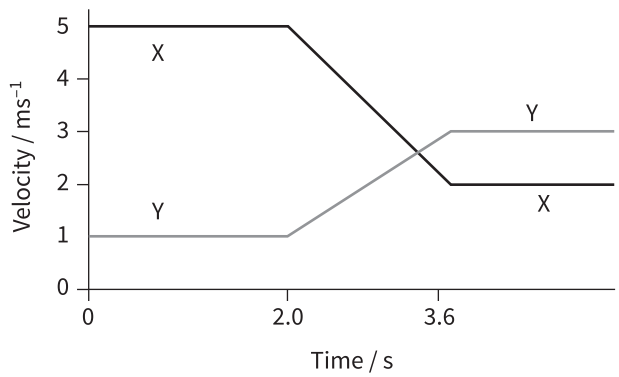


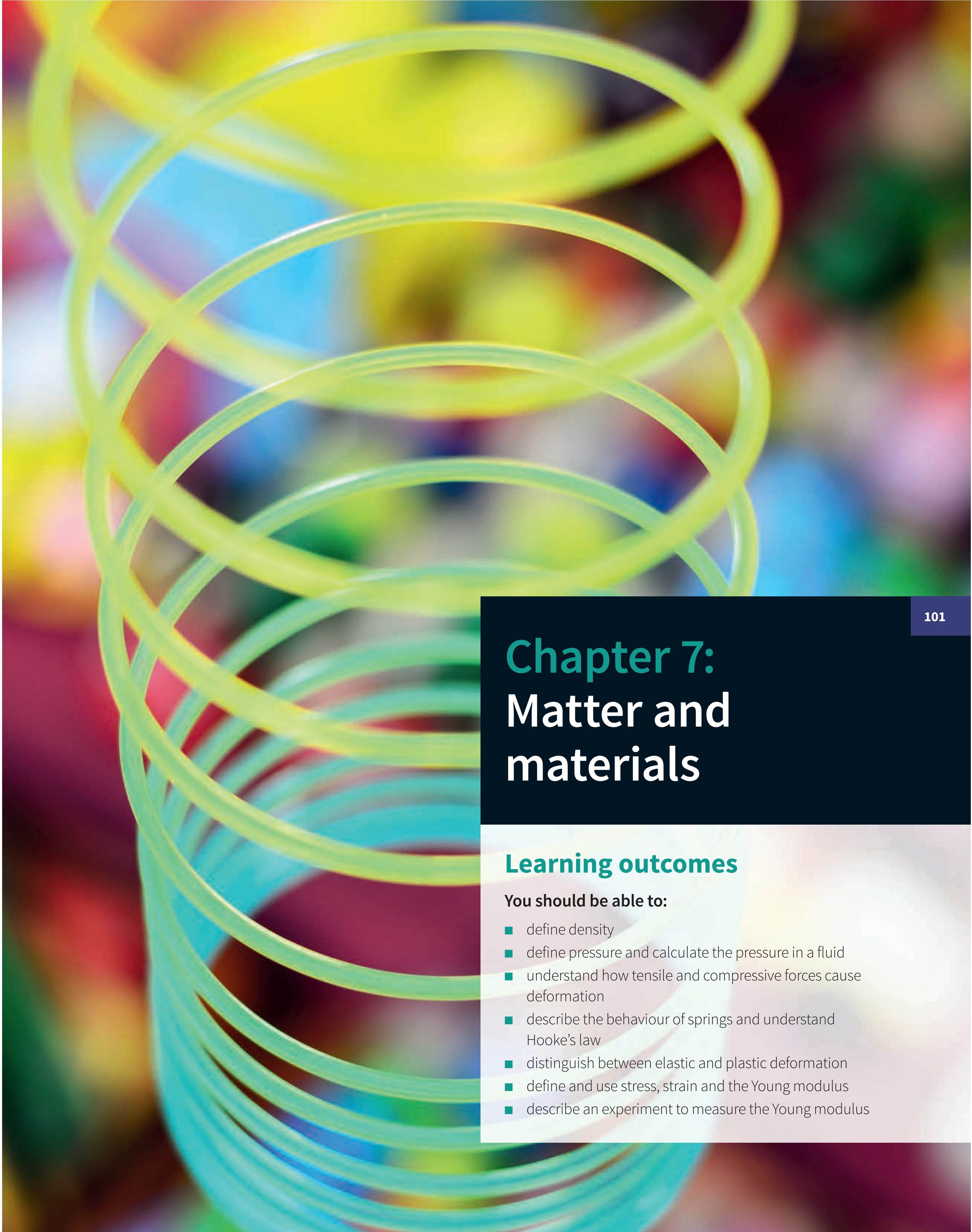
Figure 6.25 For End-of-chapter Question 11.

a Copy and complete the table. [6]

	Change in momentum / kg m s^{-1}	**Initial kinetic energy / J**	**Final kinetic energy / J**
truck X			
truck Y			

b State and explain whether the collision of the two trucks is an example of an elastic collision. [2]

c Determine the force that acts on each truck during the collision. [2]



Chapter 7: Matter and materials

Learning outcomes

You should be able to:

- define density
- define pressure and calculate the pressure in a fluid
- understand how tensile and compressive forces cause deformation
- describe the behaviour of springs and understand Hooke's law
- distinguish between elastic and plastic deformation
- define and use stress, strain and the Young modulus
- describe an experiment to measure the Young modulus

Springy stuff

In everyday life, we make great use of elastic materials. The term elastic means **springy**; that is, the material deforms when a force is applied and returns to its original shape when the force is removed. Rubber is an elastic material. This is obviously important for a bungee jumper (Figure 7.1). The bungee rope must have the correct degree of elasticity. The jumper must be brought gently to a halt. If the rope is too stiff, the jumper will be jerked violently so that the deceleration is greater than their body can withstand. On the other hand, if the rope is too stretchy, they may bounce up and down endlessly, or even strike the ground.

In this chapter we will look at how forces can change the shape of an object. Before that, we will look at two important quantities, density and pressure.

Figure 7.1 The stiffness and elasticity of rubber are crucial factors in bungee jumping.

Density

Density is a property of matter. It tells us about how concentrated the matter is in a particular material. Density is a constant for a given material under specific conditions.

Density is defined as the mass per unit volume of a substance:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{v}$$

The symbol used here for density, ρ, is the Greek letter rho.

The standard unit for density in the SI system is kg m^{-3}, but you may also find values quoted in g cm^{-3}. It is useful to remember that these units are related by:

$$1000\,\text{kg m}^{-3} = 1\,\text{g cm}^{-3}$$

and that the density of water is approximately $1000\,\text{kg m}^{-3}$.

QUESTIONS

1 A cube of copper has a mass of 240 g. Each side of the cube is 3.0 cm long. Calculate the density of copper in g cm^{-3} and in kg m^{-3}.

2 The density of steel is $7850\,\text{kg m}^{-3}$. Calculate the mass of a steel sphere of radius 0.15 m. (First calculate the volume of the sphere using the formula $V = \frac{4}{3}\pi r^3$ and then use the density equation.)

Pressure

A fluid (liquid or gas) exerts pressure on the walls of its container, or on any surface with which it is in contact. A big force on a small area produces a high pressure.

Pressure is defined as the normal force acting per unit cross-sectional area.

We can write this as a word equation:

$$\text{pressure} = \frac{\text{normal force}}{\text{cross-sectional area}}$$

$$p = \frac{F}{A}$$

Force is measured in newtons and area is measured in square metres. The units of pressure are thus newtons per square metre (N m^{-2}), which are given the special name of pascals (Pa).

$$1\,\text{Pa} = 1\,\text{N m}^{-2}$$

QUESTIONS

3 A chair stands on four feet, each of area $10\,\text{cm}^2$. The chair weighs 80 N. Calculate the pressure it exerts on the floor.

4 Estimate the pressure you exert on the floor when you stand on both feet. (You could draw a rough rectangle around both your feet placed together to find the area in contact with the floor. You will also need to calculate your weight from your mass.)

Pressure in a fluid

The pressure in a fluid (a liquid or gas) increases with depth. Divers know this: the further down they dive, the greater the water pressure acting on them. Pilots know this: the higher they fly, the lower is the pressure of the atmosphere. The atmospheric pressure we experience down here on the surface of the Earth is due to the weight of the atmosphere above us, pressing downwards. It is pulled downwards by gravity.

The pressure in a fluid depends on three factors:

- the depth h below the surface
- the density ρ of the fluid
- the acceleration due to gravity, g.

In fact, pressure p is proportional to each of these and we have:

pressure = density × acceleration due to gravity × depth

$$p = \rho g h$$

We can derive this relationship using Figure 7.2. The force acting on the shaded area A on the bottom of the tank is caused by the weight of water above it, pressing downwards. We can calculate this force and hence the pressure as follows:

volume of water = $A \times h$

mass of water = density × volume = $\rho \times A \times h$

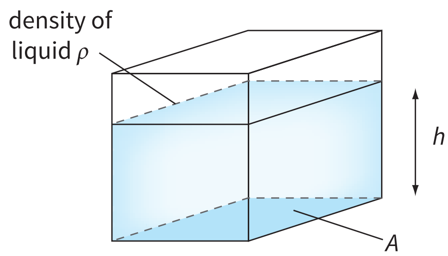


Figure 7.2 The weight of water in a tank exerts pressure on its base.

weight of water = mass × g = $\rho \times A \times h \times g$

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \rho \times A \times h \times \frac{g}{A}$$

$$= \rho \times g \times h$$

QUESTIONS

5 Calculate the pressure of water on the bottom of a swimming pool if the depth of water in the pool varies between 0.8 m and 2.4 m. (Density of water = 1000 kg m^{-3}.) If atmospheric pressure is 1.01×10^5 Pa, calculate the maximum total pressure at the bottom of the swimming pool.

6 Estimate the height of the atmosphere if atmospheric density at the Earth's surface is 1.29 kg m^{-3}. (Atmospheric pressure = 101 kPa.)

WORKED EXAMPLES

1 A cube of side 0.20 m floats in water with 0.15 m below the surface of the water. The density of water is 1000 kg m^{-3}. Calculate the pressure of the water acting on the bottom surface of the cube and the force upwards on the cube caused by this pressure. (This force is the upthrust on the cube.)

Step 1 Use the equation for pressure:

$p = \rho \times g \times h = 1000 \times 9.81 \times 0.15 = 1470$ Pa

Step 2 Calculate the area of the base of the cube, and use this area in the equation for force.

area of base of cube = 0.2 × 0.2 = 0.04 m^2

force = pressure × area = 1470 × 0.04 = 58.8 N

2 Figure 7.3 shows a manometer used to measure the pressure of a gas supply. Calculate the pressure difference between the gas inside the pipe and atmospheric pressure.

Step 1 Determine the difference in height h of the water on the two sides of the manometer.

$h = 60 - 30 = 30$ cm

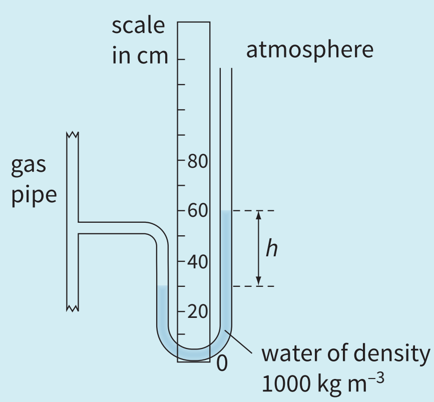


Figure 7.3 For Worked example 2.

Step 2 Because the level of water on the side of the tube next to the gas pipe is lower than on the side open to the atmosphere, the pressure in the gas pipe is above atmospheric pressure.

pressure difference = $\rho \times g \times h = 1000 \times 9.81 \times 0.30 = 2940$ Pa

Compressive and tensile forces

A pair of forces is needed to change the shape of a spring. If the spring is being squashed and shortened, we say that the forces are **compressive**. More usually, we are concerned with stretching a spring, in which case the forces are described as **tensile** (Figure 7.4).

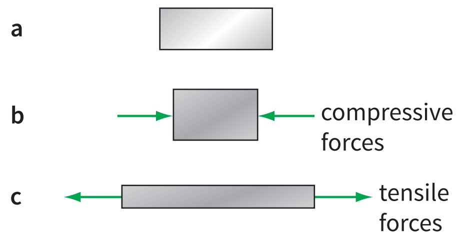


Figure 7.4 The effects of compressive and tensile forces.

When a wire is bent, some parts become longer and are in tension while other parts become shorter and are in compression. Figure 7.5 shows that the line AA becomes longer when the wire is bent and the line BB becomes shorter. The thicker the wire, the greater the compression and tension forces along its edges.

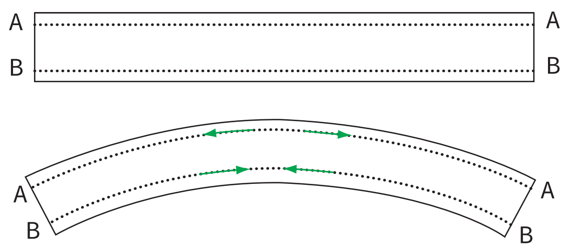


Figure 7.5 Bending a straight wire or beam results in tensile forces along the upper surface (the outside of the bend) and compressive forces on the inside of the bend.

It is simple to investigate how the length of a helical spring is affected by the applied force or load. The spring hangs freely with the top end clamped firmly (Figure 7.6). A load is added and gradually increased. For each value of the load, the extension of the spring is measured. Note that it is important to determine the increase in length of the spring, which we call the **extension**. We can plot a graph of force against extension to find the stiffness of the spring, as shown in Figure 7.7.

Hooke's law

The conventional way of plotting the results would be to have the force along the horizontal axis and the extension along the vertical axis. This is because we are changing the force (the independent variable) and this results in a change in the extension (the dependent variable). The graph shown in Figure 7.7 has extension on the horizontal axis and force on the vertical axis. This is a departure from the convention because the **gradient** of the straight section of this graph turns out to be an important quantity, known as the **force constant** of the spring. For a typical spring, the first section of this graph OA is a straight line passing through the origin. The extension x is directly proportional to the applied force (load) F. The behaviour of the spring in the linear region OA of the graph can be expressed by the following equation:

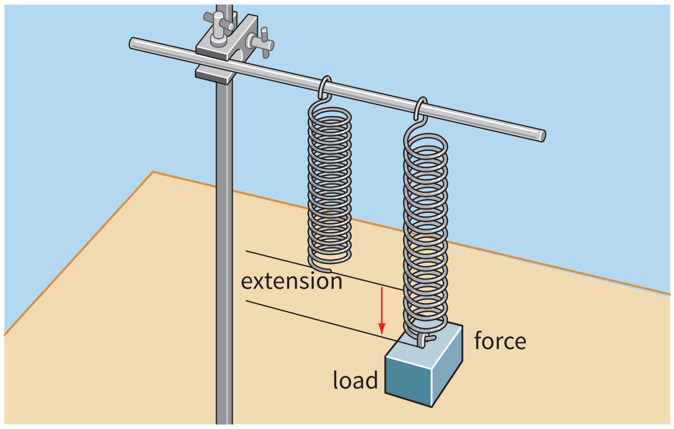


Figure 7.6 Stretching a spring.

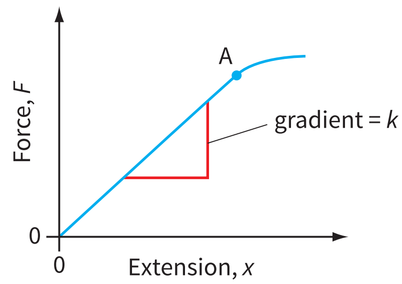


Figure 7.7 Force–extension graph for a spring.

$$x \propto F$$

or $$F = kx$$

where k is the force constant of the spring (sometimes called either the stiffness or the spring constant of the spring). The force constant is the force per unit extension. The force constant k of the spring is given by the equation:

$$k = \frac{F}{x}$$

The SI unit for the force constant is newtons per metre or $\mathrm{N\,m^{-1}}$. We can find the force constant k from the gradient of section OA of the graph:

$$k = \text{gradient}$$

A stiffer spring will have a larger value for the force constant k. Beyond point A, the graph is no longer a straight line; its gradient changes and we can no longer use the equation $F = kx$.

If a spring or anything else responds to a pair of tensile forces in the way shown in section OA of Figure 7.7, we say that it obeys **Hooke's law**:

> A material obeys Hooke's law if the extension produced in it is proportional to the applied force (load).

If you apply a small force to a spring and then release it, it will return to its original length. This behaviour is described as 'elastic'. However, if you apply a large force, the spring may not return to its original length. It has become permanently deformed. The force beyond which the spring becomes permanently deformed is known as the **elastic limit**.

QUESTION

7 Figure 7.8 shows the force–extension graphs for four springs, A, B, C and D.

- **a** State which spring has the greatest value of force constant.
- **b** State which is the least stiff.
- **c** State which of the four springs does not obey Hooke's law.

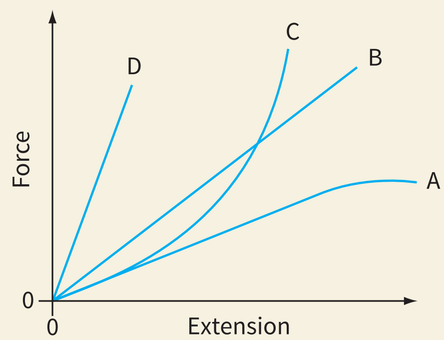


Figure 7.8 Force–extension graphs for four different springs.

Stretching materials

When we determine the force constant of a spring, we are only finding out about the stiffness of that particular spring. However, we can compare the stiffness of different **materials**. For example, steel is stiffer than copper, but copper is stiffer than lead.

Stress and strain

Figure 7.10 shows a simple way of assessing the stiffness of a wire in the laboratory. As the long wire is stretched, the position of the sticky tape pointer can be read from the scale on the bench.

BOX 7.1: Investigating springs

Springs can be combined in different ways (Figure 7.9): end-to-end (in series) and side-by-side (in parallel). Using identical springs, you can measure the force constant of a single spring, and of springs in series and in parallel. Before you do this, predict the outcome of such an experiment. If the force constant of a single spring is k, what will be the equivalent force constant of:

- two springs in series?
- two springs in parallel?

This approach can be applied to combinations of three or more springs.

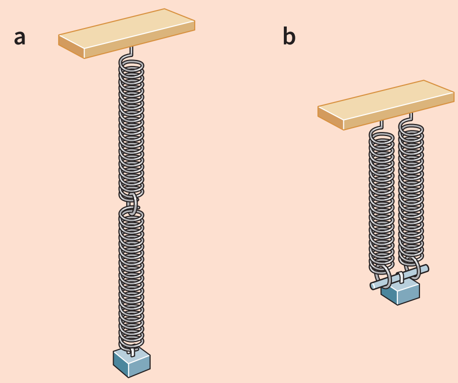


Figure 7.9 Two ways to combine a pair of springs: **a** in series; **b** in parallel.

Why do we use a long wire? Obviously, this is because a short wire would not stretch as much as a long one. We need to take account of this in our calculations, and we do this by calculating the strain produced by the load. The **strain** is defined as the fractional increase in the original length of the wire. That is:

$$\text{strain} = \frac{\text{extension}}{\text{original length}}$$

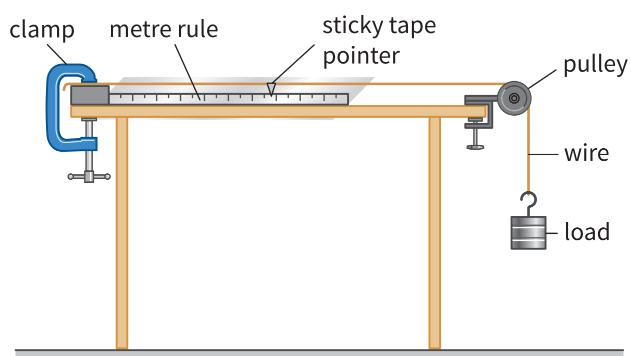


Figure 7.10 Stretching a wire in the laboratory. WEAR EYE PROTECTION and be careful not to overload the wire.